Actuarial Society of India

EXAMINATIONS

June 2005

CT5 – General Insurance, Health and Life Contingencies

Indicative Solution

Soln 1

${}_{n}q^{2}_{[x][y]}$ represents the probability that a select life, now aged y, will die within n years, having been predeceased by a select life now aged x.

[2]

Soln 2

The payments correspond to a benefit that is deferred for 10 years, then makes payments annually in advance during the remaining lifetime, up to a maximum of 25 payments. [1]

In terms of actuarial symbols, the expected present value is:

$$E[g(k)] = (D_{50} / D_{40})\, \ddot{a}_{50:25}$$ [1]

$$= (D_{50} / D_{40})\, (\ddot{a}_{50} - (D_{75} / D_{50})\, \ddot{a}_{75})$$

$$= \frac{1366.61 \times 17.444 - 363.11 \times 8.524}{2052.96}$$

$$= 10.104$$ [2]

Total [4]

Solns 3

$q_{[40]} = 0.5\, q_{40} = 0.5 \times 0.00172 = 0.00086 \Rightarrow p_{[40]} = 0.99914$ [½]

$q_{[40]+1} = 0.55\, q_{41} = 0.55 \times 0.00186 = 0.001023 \Rightarrow p_{[40]+1} = 0.99898$ [½]

$q_{[40]+2} = 0.7\, q_{42} = 0.7 \times 0.00201 = 0.001407 \Rightarrow p_{[40]+2} = 0.99859$ [½]

$q_{[40]+3} = 0.8\, q_{43} = 0.8 \times 0.00219 = 0.001752 \Rightarrow p_{[40]+3} = 0.99825$ [½]

$q_{[40]+4} = 0.9\, q_{44} = 0.9 \times 0.00240 = 0.00216 \Rightarrow p_{[40]+4} = 0.99784$ [½]

$$l_{[40]} \times p_{[40]} \times p_{[40]+1} \times p_{[40]+2} \times p_{[40]+3} \times p_{[40]+4} = l_{45}$$ [1]

⇨ $l_{[40]} \times 0.99282 = 95521$

⇨ $l_{[40]} = 96212$ [½]

Total [4]

Soln 4.

Construct a multiple decrement table

Age	No of lives	Deaths	No of withdrawals over year	No of withdrawals year end
30	100000	190	9995	8982
31	80833	153.58		

At age number of deaths = 100000*0.002*(1-0.5*0.1)= 190 [1]

No of withdrawals over the year=100000*0.1*(1-0.5*0.002)= 9995 [1]

No of withdrawals over year end =100000*(1-0.1)*(1-0.002)*0.1= 8982 [1]

Required Probability =80833*(1-0.5*0.1)*0.002= 153.58 [1]

Probability= 153.58/100000= 0.0015358 [1]

Total [5]

Soln 5

The premium is given by:

$P = 20000\, \ddot{a}^{(12)}_{67:63}$ [½]

$\ddot{a}^{(12)}_{67:63} = \ddot{a}_{67} + \ddot{a}_{63} - \ddot{a}_{67:63} - (11/24)$ [3]

$= 12.834 + 15.606 - 11.687 - 0.458$ [1]

$= 16.295$

=> P = Rs.325,900 [½]

Total [5]

Soln 6

P be the premium then maturity benefit is 1.21550625 [½]

The company invests in 5 year zero coupon bonds where maturity proceed is 1.2915479

[½]

Prob of death = $(1-l_{54}/l_{50})$= 1-91873/93925= 0.02185

[½]

Office makes no loss on death\

[½]

At t=4,office looses money if

P*1.2915479/(1+I)<1.21550625 P

[½]

or I+i>1.0625

P(1+I>1.06256) for lognormal 1+I is

[½]

P(z>(ln1.06256-0.05)/0.01)= P(z>1.07) =1-0.85769= 0.14231

[1 ½]

P(policy matures)=1-0.02185=0.97815

[½]

P(of loss)= 0.97815*0.14231=0.1392

[1]

Total [6]

Soln 7

Past service Pension

$PV= 32000 x S_{40} / S_{39} x * 18 / 60 x * {}^{z}M^{ra}_{40} / {}^{s}D_{40}$

32000*3.522 / 3.539*18 / 60*128026 / 25059

=48801

[1 ½]

Future Service Pension

$PV=32000 x S_{40} / S_{39} x * 1 / 60 x * {}^{z}\bar{R}^{ra}_{40} / {}^{s}D_{40}$

=32000*3.522 / 3.539*1 / 60*2884260 / 25059

=61091

[1 ½]

Return of accumulated contributions till date

$PV=5000 * {}^{j}M^{d}_{40} / 1.03^{20} D_{40}$

=5000 / 1.03^20*323 / 3207

=279

[1 ½]

Return of future contributions on death

$PV = 0.05*32000*S_{40}/S_{39}*{}^{sj}R^{D}_{40}/sD40$

$= 0.05*32000*3.522/3.539*16258/25059$

$= 1033$

[1 ½]

PV of total benefits = 111204

[1]

Total [7]

Solns 8

Value of benefit =

$$(1/D_{25})*[\ 1000(M_{25} - M_{30}) + 1000(D_{30}/D_{35})*(M_{35} - M_{40}) + 2000(D_{30}/D_{35})(M_{40} - M_{50})$$

$$+\ 3000(D_{30}/D_{35})*(M_{50} - M_{60} + D_{60}) + 15P*(D_{30}/D_{35})*D_{60}\ ]$$ [3]

Let P be the annual premium. Then

Value of premiums = $(1/D_{25})*[\ (N_{25} - N_{30}) + (D_{30}/D_{35})*(N_{35} - N_{45})\ ] * P$ [3]

$$P = \frac{1000\,(M_{25} - M_{30}) + (D_{30}/D_{35})[1000(M_{35} - M_{40}) + 2000(M_{40} - M_{50}) + 3000(M_{50} - M_{60} + D_{60})]}{(N_{25} - N_{30}) + (D_{30}/D_{35})*[(N_{35} - N_{45}) - 15D_{60}]}$$ [2]

$$= \frac{9480 + (3060.13/2507.40)*[8570 + 47240 + 2788890]}{17268.10 + (3060.13/2507.40)*[21079.20 - 13242.75]}$$

$$= \frac{3481264.20}{26832.01}$$

$= 129.74$ [2]

Total [10]

Solns 9

(i) The "death strain at risk" for a policy for year t+1 (t =0,1,2….) is the excess of the sum assured (S) (i.e. the present value at time t ~~+1~~ of all benefits payable on death during year t + 1) over the end of year provision ($_{t+1}V$). [1]

The "Expected death strain at risk" for year t + 1 is the total death strain that would be incurred in respect of all policies in force at the start of year t + 1 if deaths conformed to the numbers expected. [½]

EDS for year t + 1 = Ó $q\,(S - {}_{t+1}V)$ [½]

Policies in force at start of year

The "actual death at strain" for year t + 1 is the total death strain incurred in respect of all claims actually arising during year t + 1. [½]

ADS for year t + 1 = Ó $(S - {}_{t+1}V)$ [½]

Claims during Year

Total [3]

<u>"other valid definitions may also be given credit"</u>

(ii) The net premiums per unit sum insured for the 3 types of policies can be found as:

$P_a\ \ddot{a}_{45:20} = A_{45:20}$ => $P_a = \dfrac{0.46998}{13.780} = 0.03411$ [1]

$P_b\ \ddot{a}_{45:20} = A_{45:20}$ => $P_b = \dfrac{0.05923}{13.780} = 0.00430$ [1]

$P_c = P_a - P_b = 0.02981$ [1]

The net provisions at the end of the year per unit sum insured are:

${}_{10}V_a = A_{55:10} - P_a\ \ddot{a}_{55:10} = 0.68388 - 0.03411 \times 8.219 = 0.4036$ [1]

${}_{10}V_b = A^{1}_{55:10} - P_a\ \ddot{a}_{55:10} = 0.06037 - 0.00430 \times 8.219 = 0.02504$ [1]

${}_{10}V_c = (D_{65} / D_{55}) - P_c\ \ddot{a}_{55:10} = 0.62351 - 0.02981 \times 8.219 = 0.3785$ [1]

The Total expected death strain is:

$EDS = EDS_a + EDS_b + EDS_c$

$= q_{54}$ [500000(1 - ${}_{10}V_a$) + 300000 (1 - ${}_{10}V_b$) + 50000 (0 - ${}_{10}V_c$)]

= 0.003976 [298200 + 292488 – 18925]

= 2273.33 [3]

The Total actual death strain is:

ADS = $ADS_a + ADS_b + ADS_c$

= 8000(1 - ${}_{10}V_a$) + 4000 (1 - ${}_{10}V_b$) + 1000 (0 - ${}_{10}V_c$)

= 4771.2 + 3899.84 – 378.50

= 8292.54 [2]

Profit from mortality = EDS – ADS

= 2273.33 – 8292.54

= - 6019.21

i.e. there is mortality loss of Rs.6019.21 [1]

Total [12]

Solns 10

Simple bonus version:

$L = 250 + (S[1 + (0.06)K_x] + 150)\, v^{T_x} - \{P(0.98)\ddot{a}_{\min[1+K_x,\, 65-x]} + 0.02P\}$ [2]

Compound bonus version

$L = 250 + (S[1.04)^{K_x} + 150)\, v^{T_x} - \{P(0.98)\ddot{a}_{\min[1+K_x,\, 65-x]} + 0.02P\}$ [2]

Total [4]

(ii) Equivalence principle => E(L) = 0 [½]

Also assume that E[T] = E[K] + ½ [½]

Simple bonus:

⇨ $250 + (S + 150)\, A_{[x]} + (0.06S)(IA)_{[x]+1}(D_{[x]+1}) / D_{[x]}) = P[(0.98)\, \ddot{a}_{[x]:\overline{65-x}|} + 0.02]$ [1]

⇨ $250 + (S + 150)\,A_{[x]} + (0.06S)(IA)_{[x]+1}(D_{[x]+1}) / D_{[x]})$ can be more easily valued as

⇨ $250 + (0.94S + 150)\,A_{[x]} + 0.06S(IA)_{[x]}$ [½]

In this case:

$250 + (1.04)^{½} [\{(0.94)(200000) + 150\}\,A_{[40]} + (R_{[40]}) / (D_{[40]})\ (0.06)(200000)]$

$= P[(0.98)\,ä_{[40]:25} + 0.02]$ [½]

⇨ $250 + (1.04)^{½} [188150\,(0.23041) + (12000)\,(16334.87 / 2052.54)]$

$= P[(0.98)(15.887) + 0.02]$

⇨ $250 + (1.04)^{½} [43351.64 + 95500.42] = P[15.58926]$

⇨ $P = 141851.88 \div 15.58926 =$ Rs.9099 p.a. [1]

Compound bonus:

$250 + (1.04)^{½} [(A^{*}_{[40]} / 1.04)\,200000 + 150\,A_{[x]}] = 15.58926\,P$

* at $[(i - b) / (1 + b)]$ i.e. 0% [1]

⇨ $(250) + (1.04)^{½} [(200000 / 1.04) + (150)(0.23041)] = 15.58926\,P$ [½]

⇨ $250 + (1.04)^{½} [192307.69 + 34.562] = 15.58926\,P$

⇨ $P = 196401.38 \div 15.58926 =$ Rs.12599 p.a. [½]

Total [6]

(iii) Net Premium Provision for WP policies

1) allows for accrued bonuses only [½]
2) net premium ignoring any bonuses [½]

⇨ ${}_{10}V = 290000\,A_{50} - (NP)\,ä_{50:15}$ [1]

⇨ where $NP = 200000\,(A_{40} / ä_{40:25}) = [(200000)\,(1.04)^{½}\,(0.23056)] / [15.884]$

⇨ $= 2960.54$ p.a. [1]

⇨ ${}_{10}V = (290000)\,(1.04)^{½}\,(0.32907) - (2960.54)\,(11.253)$ [1]

⇨ $= 97320.19 - 33314.957 = 64005.23$ [1]

Total [5]

Total for Qn. [12]

Soln 11.

Section A			95%	101%	
Unit Fund (ignoring actuarial funding)			1	2	3
Value of capital units at start	(1)	Carried over	0.00	3963.79	4093.70
Premium to CU	(2)		3838.00	0.00	0.00
Interest on CU	(3)	=[(1)+(2)]*(1+i)	345.42	356.74	368.43
Management charge on CU	(4)	=(3)*m	219.63	226.83	234.26
Nominal Value of capital units at end	(5)	=+(1)+(2)+(3)-(4)	3963.79	4093.70	4227.87
Accumulation Fund			1.00	2.00	3.00
Value of acc units at start	(1)	Carried over	0.00	0.00	4131.13
Premium to AU	(2)		0.00	3838.00	3838.00
Interest on AU	(3)	=[(1)+(2)]*(1+i)	0.00	345.42	717.22
Management charge on AU	(4)	=(3)*m	0.00	52.29	108.58
Value of acc units at end	(4)	=+(1)+(2)+(3)-(4)	0.00	4131.13	8577.77
Total Unit reserve			3963.79	8224.83	12805.64
Surrender Value of units			3369.22	7815.46	12805.64
Section B					
Sterling Fund					
Unallocated Premium	(1)		162.00	162.00	162.00
Expenses	(2)		500.00	100.00	100.00
Interest	(3)	=[(1)+(2)]*(1+i)	-15.21	2.79	2.79
Management charge on CU	(4)	From Section A above	219.63	226.83	234.26
Management Charge on AU	(5)	From Section A above	0.00	52.29	108.58
Pols in force at end of year before surrender	(6)	1-qx	0.989888	0.988656	0.987284
No of pols surrendering	(7)	=15%*(6)	0.148483	0.148298	0.000000
No of pols in force at end of year after surrender	(8)	=(6)-(7)	0.841405	0.840358	0.987284

Surrender Profit	(9)	(Surrender value-Actual CU+AU)*(7)	88.28	60.71	0.00
Extra death benefit	(10)	(10000-Actual CU-Actual AU)*(1-(1))	61.04	20.14	0.00
End of year cashflow	(11)	(1)-(2)+(3)+(4)+(5)+(9)-(10)-(11)	-106.34	279.90	190.47
Probability in force year beginning			1.00000	0.84140	0.70708
Prob of death			0.01011	0.00954	0.00899
Prob of surrender			0.14848	0.12478	0.00000
Policies in force year end			0.84140	0.70708	0.69809
Discount Factor			0.86957	0.75614	0.65752
Expected PV of cash flow			-92.47	178.08	88.55
Expected PV of Premiums			4000.00	2926.63	2138.62
Expected PV			174.17		
Expected PV of premiums			9065.24		
Profit Margin			1.92%		
Revised Sterling fund					
Unallocated Premium			162.00	162.00	162.00
Expenses			500.00	100.00	100.00
Interest			-15.21	2.79	2.79
MC on capital units			219.63	226.83	234.26
MC on acc units			0.00	52.29	108.58
Surrender profit			88.28	60.71	0.00
Extra death benefit			61.04	20.14	0.00
End of year cash flow			-106.34	384.48	407.63
Reserves at start of year per policy			400.00	400.00	400.00
Intrest on reserves			18.00	18.00	18.00
Reserves at year end for policies in force			336.56	336.14	0.00
Change in reserve at year end			-63.44	-63.86	-400.00
Revised cash flow			-424.90	66.34	425.63
Discount Factor			0.87	0.76	0.66
Expected Present value			-369.48	42.21	197.88
Expected present value of profit			-129.39		
Profi Margin			-1.43%		

iii) the cost of extra death benefit would decrease and separately the profit signature will increase. The effect of these two factors would be to increase the NPV of profit in part(i) and part ii)

[2]

Total [27]