Actuarial Society of India

EXAMINATIONS

15th June 2005

Subject CT4 (104) – Models (104 Part)

Time allowed: One and a Half Hours (10.30 am – 12.00 noon)

INSTRUCTIONS TO THE CANDIDATES

1. ***Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***
5. ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor

Q.1 T_x is the future life time random variable of a person of age x; express

m_{x+t} : force of mortality

$f_x(t)$ probability density function of T_x

in terms of probability limits of T_x. **[2]**

Q.2 a) Assuming uniform distribution of deaths between age x and x+1, show that

(i) $m_{x+t} = \dfrac{q_x}{1-tq_x}$ **[1]**

(ii) ${}_{t-s}q_{x+s} = \dfrac{(t-s)q_x}{1-sq_x}$ for $s+t \le 1$ **[2]**

b) Given $p_{56} = 0.990581$ $p_{57} = 0.989503$ and $p_{58} = 0.988314$

Find ${}_2p_{56.75}$ assuming

(i) A uniform distribution of deaths between integral ages. **[2]**

(ii) A constant force of mortality between integral ages **[2]**

Total [7]

Q.3 For a group term assurance business, the actual mortality experience of the group is being considered for incorporation into its pricing (experience rating). A mortality rate of 0.002 is assumed for the group.
Find the minimum size of the group to be fully credible where full credibility means that the number of deaths is within 5% of the mean, 95% of the time. Assume that the number of deaths for the scheme follows a typical Poisson distribution. **[4]**

Q.4 In a mortality investigation the following data has been recorded for 7 independent lives observed between exact ages 40 and 41 years.

s_i = the time in years after exact age 40 when the i^{th} life came under observation.

t_i = time in years after exact age 40 when the i^{th} life was censored.

d_i = 1 if i^{th} life died before x + t_i

= 0 if i^{th} life survived to x + t_i

u_i : if d_i =1, then x + u_i is the age at death.

The data observed during the investigation is set out in the table below:

i	s_i	t_i	d_i	u_i
1	0	1	0	-
2	0.4	0.7	0	-
3	0.3	0.9	1	0.7
4	0	0.9	1	0.6
5	0	1	1	0.8
6	0.5	1	1	0.9
7	0	0.4	0	-

a) **(i)** Under Binomial model of mortality, write down the likelihood of these observations.

(ii) Assuming uniform distribution of death over age (40,41) express this likelihood in terms of q_{40}. **[4]**

b) Using Poisson model of mortality and assuming constant force of mortality m'_{40} over (40,41), write down the likelihood of these observations. **[3]**

Hence calculate the maximum likelihood estimate of m'_{40} **[3]**

Total [10]

Q.5 The table below gives the data for a small sample of heart patients in a hospital. It shows the time in months until death. Observations marked * show that the patient either left the hospital or died due to a cause not related to heart condition.

Males	5*	10	12*	14	15*	18*	19	
Females	1*	3	6	7*	9*	11*	16	20*

A Cox partial hazard model

$l(t|x) = l_0(t)e^{bx}$ is to be fitted to these data

where t = time till death

$l_0(t)$ = baseline hazard

x = 0 for males , x = 1 for females

(i) Write down the general expression for the partial likelihood for such investigation **[1]**

(ii) Derive an expression for the partial likelihood for the above data **[5]**

(iii) Calculate the maximum partial likelihood estimate of b. **[5]**

(iv) We are subsequently able to generate the following additional data

Male 19

Female 8*

Write down the partial likelihood after including the additional data provided. **[3]**

Total [14]

Q.6 **a)** In a mortality investigation, the total number of deaths at age x during the period of investigation is q_x. Age x is defined as

Age last birthday at start of the policy + curtate duration at date of death.

(i) State the rate interval implied by this definition. **[1]**

(ii) Give the age x + f applicable for the estimates of mortality q_x and m_x. State all assumptions used in determining f. **[2]**

(iii) How would your answer to (ii) above change if we assume that, on average, policies were purchased 3 months before the birthdays. **[2]**

b) Two life companies are carrying out mortality investigations. Each company is using census method with the same definition of age but deaths are tabulated differently.

For both companies the census of the number of policyholders at 1.1.2000 and 1.1.2001 are available tabulated by age nearest birthday at entry plus curtate duration at census date.

The deaths during 2000 are tabulated as

Company I : by age nearest birthday at death

Company II : by age nearest birthday on 1.1.2000

(i) Derive with reasoning, a suitable formula for calculating the exposed to risk in each case so that the central rates of mortality can be calculated.

[6]

(ii) Give the age at which the observed rates of mortality apply in each case. State assumptions that needs to be made.

[2]

Total [13]

****