# Actuarial Society of India

# EXAMINATIONS

15th June 2005

## Subject CT4 – Models

**Time allowed: Three Hours (10.30 am – 1.30 pm)**

***INSTRUCTIONS TO THE CANDIDATES***

1. ***Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Please do not write answers of subject 103 in subject 104 answersheets and vice versa.***
5. ***Fasten your answer sheets of subject 103 and 104 in numerical order of questions. This, you may complete immediately after expiry of the examination time.***
6. ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

**Professional Conduct:**

*"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."*

**AT THE END OF THE EXAMINATION**

**Hand in both your answer scripts and this question paper to the supervisor**

# 103 Questions

**Q.A1**

Two trains 2301 and 2401 stop at Dadar and go to Churchgate. I am waiting at Dadar. 2301 is a "fast" train and has fewer stops and hence takes lesser time. Both the trains arrive at Dadar in accordance with independent Poisson processes with parameters 5 per hour and 1 per hour respectively.

**i)** Calculate the probability that

**a)** Exactly six 2301's and no 2401 to arrive in the next one hour

**b)** At least 3 trains arrive in the next half an hour

**c)** I have to wait for more than 15 minutes for a train, if I have just missed one

**d)** I see exactly one 2301 pass while I am waiting for a 2401

**[10]**

**ii)** Suppose I need to get to a stop that only 2301 services and not 2401. If I forget this fact and just take the first train that arrives, what is the probability that I catch the wrong train. **[2]**

**iii)** If half the 2401's and a third of the 2301's are manufactured at Chittaranjan, independently of anything else, how long do I have to wait for a Chittaranjan manufactured train, if one hasn't arrived for over an hour. **[3]**

**Total [15]**

**Q.A2**

For a discrete time stochastic process $X_n$ , define the terms

- Stationary
- Weakly stationary
- Increment
- Markov property
- Martingale

**Total [10]**

**Q.A3**

A fair coin is tossed repeatedly. Every time it lands heads, Aishwarya pays Vivek Re. 1 and every time it hands tails, Vivek pays Aishwarya Re. 1. Vivek is assumed to have access to infinite resources (essentially assumed to be very rich).

Let Aishwarya's available funds at time n be denoted by $S_n$. Assume that she starts with Rs. k and that if she goes broke she will have to stop playing but if she reaches Rs. K she will quit while ahead; where $0<k<K$.

**a)** What process that $S_n$ follow? What is the initial condition? What are the boundary conditions? **[3]**

**b)** Show that $S_n$ is a Martingale. **[2]**

**c)** Define the term stopping time. State the Optimal Stopping Theorem. **[4]**

**d)** Find the probability that Aishwarya is ultimately ruined. **[3]**

**e)** If Aishwarya gets greedy and does not quit while ahead, what is her ultimate probability of ruin. **[2]**

**f)** Why have we assumed that Vivek is very rich? **[1]**

**Total [15]**

**Q.A4** $B_t$ is a standard Brownian Motion with $B_0 = 0$

**a)** Calculate $P(B_1 \geq 1)$ and $P(B_2 \geq 1)$ **[3]**

**b)** Use reflection principle to find the distribution of Z = max( $B_t$ ) for $0 \leq t \leq 2$ . **[4]**

**c)** Let $X_t = B_t + mt(m > 0)$ and let $T_x$ be the first time the process $X_t$ hits x. Find an expression for $E(T_x)$ when x is 0. **[3]**

**Total [10]**

# 104 Questions

**Q.B1** $T_x$ is the future life time random variable of a person of age x; express

$\mu_{x+t}$ : force of mortality

$f_x(t)$ probability density function of $T_x$

in terms of probability limits of $T_x$. **[2]**

**Q.B2 a)** Assuming uniform distribution of deaths between age x and x+1, show that

**(i)** $\mu_{x+t} = \dfrac{q_x}{1-tq_x}$ **[1]**

**(ii)** ${}_{t-s}q_{x+s} = \dfrac{(t-s)q_x}{1-sq_x}$ for $s+t \le 1$ **[2]**

**b)** Given $p_{56} = 0.990581$ $\quad p_{57} = 0.989503$ and $\quad p_{58} = 0.988314$

Find ${}_2p_{56.75}$ assuming

**(i)** A uniform distribution of deaths between integral ages. **[2]**

**(ii)** A constant force of mortality between integral ages **[2]**

**Total [7]**

**Q.B3** For a group term assurance business, the actual mortality experience of the group is being considered for incorporation into its pricing (experience rating). A mortality rate of 0.002 is assumed for the group.
Find the minimum size of the group to be fully credible where full credibility means that the number of deaths is within 5% of the mean, 95% of the time. Assume that the number of deaths for the scheme follows a typical Poisson distribution. **[4]**

**Q.B4** In a mortality investigation the following data has been recorded for 7 independent lives observed between exact ages 40 and 41 years.

$s_i$ = the time in years after exact age 40 when the $i^{th}$ life came under observation.

$t_i$ = time in years after exact age 40 when the $i^{th}$ life was censored.

$d_i$ = 1 if $i^{th}$ life died before x + $t_i$

= 0 if $i^{th}$ life survived to x + $t_i$

$u_i$ : if $d_i$ =1, then x + $u_i$ is the age at death.

The data observed during the investigation is set out in the table below:

| **i** | $s_i$ | $t_i$ | $d_i$ | $u_i$ |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | - |
| 2 | 0.4 | 0.7 | 0 | - |
| 3 | 0.3 | 0.9 | 1 | 0.7 |
| 4 | 0 | 0.9 | 1 | 0.6 |
| 5 | 0 | 1 | 1 | 0.8 |
| 6 | 0.5 | 1 | 1 | 0.9 |
| 7 | 0 | 0.4 | 0 | - |

**a)** **(i)** Under Binomial model of mortality, write down the likelihood of these observations. **[4]**

**(ii)** Assuming uniform distribution of death over age (40,41) express this likelihood in terms of $q_{40}$.

**b)** Using Poisson model of mortality and assuming constant force of mortality $\mu'_{40}$ over (40,41), write down the likelihood of these observations. **[3]**

Hence calculate the maximum likelihood estimate of $\mu'_{40}$ **[3]**

**Total [10]**

**Q.B5** The table below gives the data for a small sample of heart patients in a hospital. It shows the time in months until death. Observations marked * show that the patient either left the hospital or died due to a cause not related to heart condition.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Males | 5* | 10 | 12* | 14 | 15* | 18* | 19 | |
| Females | 1* | 3 | 6 | 7* | 9* | 11* | 16 | 20* |

A Cox partial hazard model

$\lambda(t \mid x) = \lambda_0(t)e^{\beta x}$ is to be fitted to these data

where t = time till death

$\lambda_0(t)$ = baseline hazard

x = 0 for males , x = 1 for females

**(i)** Write down the general expression for the partial likelihood for such investigation **[1]**

**(ii)** Derive an expression for the partial likelihood for the above data **[5]**

**(iii)** Calculate the maximum partial likelihood estimate of $\beta$. **[5]**

**(iv)** We are subsequently able to generate the following additional data

Male 19

Female 8*

Write down the partial likelihood after including the additional data provided. **[3]**

**Total [14]**

**Q.B6 a)** In a mortality investigation, the total number of deaths at age x during the period of investigation is $\theta_x$. Age x is defined as

Age last birthday at start of the policy + curtate duration at date of death.

**(i)** State the rate interval implied by this definition. **[1]**

**(ii)** Give the age x + f applicable for the estimates of mortality $q_x$ and $m_x$. State all assumptions used in determining f. **[2]**

**(iii)** How would your answer to (ii) above change if we assume that, on average, policies were purchased 3 months before the birthdays. **[2]**

**b)** Two life companies are carrying out mortality investigations. Each company is using census method with the same definition of age but deaths are tabulated differently.

For both companies the census of the number of policyholders at 1.1.2000 and 1.1.2001 are available tabulated by age nearest birthday at entry plus curtate duration at census date.

The deaths during 2000 are tabulated as

Company I : by age nearest birthday at death

Company II : by age nearest birthday on 1.1.2000

**(i)** Derive with reasoning, a suitable formula for calculating the exposed to risk in each case so that the central rates of mortality can be calculated.
**[6]**

**(ii)** Give the age at which the observed rates of mortality apply in each case. State assumptions that needs to be made.
**[2]**

**Total [13]**

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