Actuarial Society of India

EXAMINATIONS

15th June 2005

Subject CT4 (103) – Models (103 Part)

Time allowed: One and a Half Hours (10.30 am – 12.00 noon)

INSTRUCTIONS TO THE CANDIDATES

1. *Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all questions, beginning your answer to each question on a separate sheet.*

4. *Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.*

5. *In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.*

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor

Q.1   Two trains 2301 and 2401 stop at Dadar and go to Churchgate. I am waiting at Dadar. 2301 is a "fast" train and has fewer stops and hence takes lesser time. Both the trains arrive at Dadar in accordance with independent Poisson processes with parameters 5 per hour and 1 per hour respective ly.

i)   Calculate the probability that

a) Exactly six 2301's and no 2401 to arrive in the next one hour
b) At least 3 trains arrive in the next half an hour
c) I have to wait for more than 15 minutes for a train, if I have just missed one
d) I see exactly one 2301 pass while I am waiting for a 2401

[10]

ii)   Suppose I need to get to a stop that only 2301 services and not 2401. If I forget this fact and just take the first train that arrives, what is the probability that I catch the wrong train.   [2]

iii)   If half the 2401's and a third of the 2301's are manufactured at Chittaranjan, independently of anything else, how long do I have to wait for a Chittaranjan manufactured train, if one hasn't arrived for over an hour.   [3]

Total [15]

Q.2   For a discrete time stochastic process X_n , define the terms
- Stationary
- Weakly stationary
- Increment
- Markov property
- Martingale

Total [10]

Q.3   A fair coin is tossed repeatedly. Every time it lands heads, Aishwarya pays Vivek Re. 1 and every time it hands tails, Vivek pays Aishwarya Re. 1. Vivek is assumed to have access to infinite resources (essentially assumed to be very rich). Let Aishwarya's available funds at time n be denoted by S_n. Assume that she starts with Rs. k and that if she goes broke she will have to stop playing but if she reaches Rs. K she will quit while ahead; where 0<k<K.

a) What process that S_n follow? What is the initial condition? What are the boundary conditions?   [3]
b) Show that S_n is a Martingale.   [2]
c) Define the term stopping time. State the Optimal Stopping Theorem.   [4]
d) Find the probability that Aishwarya is ultimately ruined.   [3]
e) If Aishwarya gets greedy and does not quit while ahead, what is her ultimate probability of ruin.   [2]
f) Why have we assumed that Vivek is very rich?   [1]

Total [15]

Q.4 B_t is a standard Brownian Motion with $B_0 = 0$

a)   Calculate $P(B_1 \geq 1)$ and $P(B_2 \geq 1)$ [3]

b)   Use reflection principle to find the distribution of $Z = \max(B_t)$ for $0 \leq t \leq 2$.   [4]

c)   Let $X_t = B_t + mt (m>0)$ and let T_x be the first time the process X_t hits x. Find an expression for $E(T_x)$ when x is 0.   [3]

Total [10]

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