Institute of Actuaries of India

Subject CT3 – Probability & Mathematical Statistics

April 2016 Examination

Indicative Solution

Introduction:

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other approaches leading to a valid answer and examiners have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

Average claims, $\bar{x} = \frac{\sum fixi}{\sum fi}$ = (1(5) + 2(30) + 3(45) + 4(10) + 5(10)) / 100
= 290 / 100 = 2.90 claims

Variance of claims = $\frac{(\sum fixi^{2} - n\bar{x}^2)}{(n-1)}$

Where $\sum fixi^{2}$ = 940

Hence, Variance = $\frac{(940 - 100 * 2.90^2)}{99}$ = 99/99 = 1

Median of number of claim is the (100+1)/2 = 50.5th observation i.e. average of 50th and 51st observations = $\frac{3+3}{2}$ = 3

Mode of the number of claims = the highest frequency = 3

Range of number of claims= 5-1 =4

[5 Marks]

Solution 2:

Given that P (A) = 0.3; P (B) = 0.6; P (A | B) = 0.4
P ($A^c \cap B^c$) = P ((A U B)^c) = 1 – P (A U B)

Now P (AUB) = P (A) + P (B) – P (A ∩ B)

Whereas P (A ∩ B) = P (B) P (A | B) = 0.6 (0.4) =0.24

Hence P (A U B) = 0.3 + 0.6 - 0.24 = 0.66

P ($A^c \cap B^c$) = 1 - 0.66 = 0.34

[3 Marks]

Solution 3:

i)

Let O: Event that a randomly selected Indian adult is Obese
H: Event that a randomly selected Indian adult has Hypertension

Given P (H | O) = 2/5; P (H | O’) = 1/10; P (O) = 3/10

a)

P (O ∩ H) = P (H ∩ O) = P (H | O) P (O) = (2/5) (3/10) = 6/50 = 0.12 [1] b)

$P(O \cap H') = P(H' \cap O)$
$= P(H' \mid O)\, P(O)$
$= (1 - P(H \mid O))\, P(O)$
$= (1-2/5)(3/10) = (3/5)(3/10)$
$= 9/50 = 0.18$ [1]

c)

$P(O' \cap H) = P(H \cap O')$
$= P(H \mid O')\, P(O')$
$= (1/10)(7/10)$
$= 7/100 = 0.07$ [1]

d)

$P(O' \cap H') = P(H' \cap O')$
$= P(H' \mid O')\, P(O')$
$= [1 - P(H \mid O')]\,[1 - P(O)]$
$= (1-1/10)(1-3/10)$
$= (9/10)(7/10) = 63/100 = 0.63$ [1]

Alternate Solution:
We have to find $P(O' \cap H')$

$P(O' \cap H') = [P(O \cup H)]' = 1 - P(O \cup H)$
$= 1 - [P(O) + P(H) - P(O \cap H)]$

Now, from (i) above, $P(O \cap H) = 0.12$ *and* $P(O) = 0.30$ *(given)*

Now to find $P(H)$

$P(H) = P(O \cap H) + P(O' \cap H)$

From (i) above, $P(O \cap H) = 0.12$
Now, from (iii) above, $P(O' \cap H) = 0.07$

So,
$P(H) = 0.12 + 0.07 = 0.19$

Hence,
$P(O' \cap H') = 1 - [0.30 + 0.19 - 0.12] = 1 - (0.37) = 0.63$

e)

P (H) = P (H ∩ O) + P (H ∩ O')

From (i) and (iii) above we get: P (H) = (6/50) + (7/100) = 19/100 = 0.19 [1]

ii)

The above results are summarized in the table of joint probabilities

	Without Hypertension	With Hypertension	Total
Non-Obese	0.63	0.07	0.70
Obese	0.18	0.12	0.30
Total	0.81	0.19	1.00

The yearly expected cost of medicines

=2000 (0.07) 1000 + 4000 (0.12) 1000 = 1, 40, 000 + 4, 80, 000 = Rs 6, 20, 000

Club will incur Rs 6, 20, 000 as yearly expected cost of medicines. [3]

[8 Marks]

Solution 4:

i)

Suppose that there are *m* items in sample *A* and *n* items in sample *B.* Then, from the information given

$\overline{\mathrm{A}} = \frac{\sum \mathrm{A}}{\mathrm{m}} = 10 => \sum \mathrm{A} = 10\mathrm{m}$

$\overline{\mathrm{B}} = \frac{\sum \mathrm{B}}{\mathrm{n}} = 18 => \sum \mathrm{B} = 18\mathrm{n}$

And m + n = 40, so $\frac{\sum \mathrm{A} + \sum \mathrm{B}}{40} = 16 => 10m + 18n = 640$

Solving the simultaneous equations gives m=10 and n=30 [2]

ii)

Now consider the sample variances. We have:

$30 = \frac{1}{9}\{\sum \mathrm{A}^2 - 10\,(10)^2\} => \sum \mathrm{A}^2 = 1270$

$72 = \frac{1}{29}\{\sum \mathrm{B}^2 - 30\,(18)^2\} => \sum \mathrm{B}^2 = 11808$

So the combined variance is given by:

$\mathrm{S}^2 = \frac{1}{39}\{(1270 + 11808) - 40\,(16)^2\} = 72.77$ [4]

iii)

The combined variance is bigger than the variance of either of the original samples. This is because the two samples are clustered in different areas (A around mean = 10 and B around mean = 18), so that by combining the samples we obtain more overall variation

[1]

[7 Marks]

Solution 5:

Let V = Max (X1, X2,…,Xn)

F_V (x) = P (V < x) = P (Max (X1, X2,…,Xn) < x)

= P (X1 < x, X2 < x, …,Xn < x)

= $\mathrm{P}\ (\mathrm{X} < x)^n$

Now P (X < x) =$\int_0^{\mathrm{x}} \lambda\, \mathrm{e}^{-\lambda \mathrm{t}}$ dt = 1- $\mathrm{e}^{-\lambda \mathrm{x}}$

Hence, F_V (x) = P (V < x) = $(1 - \mathrm{e}^{-\lambda\mathrm{x}})^n$

f_V (x) = F'_V (x) = n $(1 - \mathrm{e}^{-\lambda\mathrm{x}})^{(n-1)}$ (- $\mathrm{e}^{-\lambda\mathrm{x}}$) (- λ)

f_V (x) = n λ $\mathrm{e}^{-\lambda\mathrm{x}}$ $(1 - \mathrm{e}^{-\lambda\mathrm{x}})^{(n-1)}$

[4 Marks]

Solution 6:

Given u has Uniform U (a, b)

E (u) = (a + b) / 2

V (u) = (b - a) ^{2} / 12

Now, X | U is exp $\left(\frac{1}{u}\right)$

Hence, E (X | U) = u and Var (X | U) = u^2

E (X) = E (E (X | U)) = E (u) = (a + b) / 2

Var (X) = E (Var (X | U)) + Var (E (X | U))

= E (u^2) + Var (u)

= {Var (u) + [E (u)] ^{2}} + Var (u)

= {(b - a) ^{2} / 12 + (b + a) ^{2} / 4} + (b - a) ^{2} / 12

= (b - a) ^{2} / 6 + (b + a) ^{2} / 4

= (5 (a^2 + b^2) + 2ab) / 12

[6 Marks]

Solution 7:

Let X be the number of employees who continue in the organization for at least 5 years

X is Binomial (100, 0.20)

So approximately X is N (20, 16)

We require P (X > 25)

Using continuity correction:

P (X > 25.5) = P (Z > (25.5-20)/4)

= P (Z > 1.375) = 1- P (Z < 1.375)

= 1-0.915435 =0.084565 **[4 Marks]**

OR

Here we can approximate it by Poisson distribution as Poisson (20)

P(X>25.5) = P (Z > (25.5-20)/$\sqrt{20}$) = P (Z > 1.23)

= 1 - P (Z < 1.23) = 1 - 0.89065 =0.10935

(If the candidate answers using Poisson approximation only 3 marks may be awarded)

Solution 8:

i)

Let $\overline{X}_n$ denote the average tip amount based on n tips. From the Central Limit Theorem (CLT) that if the number of tips is sufficiently large then

$\overline{X}_n \sim N\left(100, \frac{10^2}{n}\right)$

We need to find n such that P ($\overline{X}_n$ > 98) > 0.95

$P\left(\frac{\overline{X}_n - 100}{\frac{10}{\sqrt{n}}} > \frac{98 - 100}{\frac{10}{\sqrt{n}}}\right) = P\left(Z > -\frac{\sqrt{n}}{5}\right) > 0.95 \Rightarrow P\left(Z < \frac{\sqrt{n}}{5}\right) > 0.95$

$\Rightarrow \frac{\sqrt{n}}{5} = 1.645$

n = 67.65, so n ~ 68 tips are required. [3]

ii)

Let X_{64} = total amount received from 64 tips on a particular day

E (X_{64}) = 64 (100) = 6400

S.D. (X_{64}) = $\sqrt{64}$ (10) = 80

P (X_{64} > 6500) = P (Z > (6500 - 6400) / 80) = P (Z > 1.25)

= 1- P (Z < 1.25) = 1 - 0.89435 = 0.10565 [3]

[6 Marks]

Solution 9:

Let the random variables X and Y be defined as

X: Baby boy's weight at birth

Y Baby girl's weight at birth

W = X - Y has a normal distribution with mean = – 500 grams; and

Standard Deviation is SD (X - Y) = [V (X - Y)] $^{0.5}$ = [V (X) + V (Y)] $^{0.5}$

SD (X - Y) = (90000 + 160000) $^{0.5}$ = 500 grams

P (X > Y) = P (X – Y > 0) = $P\left(\frac{(X-Y) - (-500)}{500} > \frac{0 - (-500)}{500}\right)$ = P (Z > 1)

= 1 - P (Z ≤ 1)

= 1 - 0.84134 = 0.15866

[3 Marks]

Solution 10:

Let D: Event that the device is defective
Given P (A) = 80%; P (B) = 15%

P (D | A) = 4%; P (D | B) = 6%; P (D | C) = 9%

P(C) = 5% as P (A) + P (B) + P (C) = 1

To find P (A | D)

$$P(A \mid D) = \frac{P(D \mid A)\,P(A)}{P(D \mid A)\,P(A) + P(D \mid B)\,P(B) + P(D \mid C)\,P(C)}$$

$$= \frac{4\%(80\%)}{4\%(80\%) + 6\%(15\%) + 9\%(5\%)} = 0.7033$$

[3 Marks]

Or

Let us assume that there are 100 devices

Company	*Defective*	*Non-Defective*	*Total*
A	*3.20*	*76.80*	*80*
B	*0.90*	*14.10*	*15*
C	*0.45*	*4.55*	*5*
Total	*4.55*	*95.45*	*100*

P (A | D) = 3.20 / 4.55 = 0.7033

Solution 11:

i)

Comparing the distribution of X from the Actuarial tables;
X is Gamma $\left(10, \frac{1}{\beta}\right)$ [1]

ii)

$$f(x) = \frac{x^9 e^{\left(\frac{-x}{\beta}\right)}}{\beta^{10} 9!}$$

The likelihood L is: $L = \prod_{i=1}^{n} f(x_i) = \frac{\prod_{i=1}^{n} x_i{}^9 e^{\left(\frac{-x_i}{\beta}\right)}}{\beta^{10} 9!}$

Taking logs, we get:

$l = logL = 9\sum_{i=1}^{n} \log x_i - \frac{\sum_{i=1}^{n} x_i}{\beta} - 10nlog\beta - nlog(9!)$

Differentiating with respect to β:

$\frac{\partial l}{\partial \beta} = \frac{\sum_{i=1}^{n} x_i}{\beta^2} - \frac{10n}{\beta}$

Setting this equal to zero gives:

$\hat{\beta} = \frac{\sum_{i=1}^{n} x_i}{10n}$

Checking for maximum:

$\frac{\partial^2 l}{\partial \beta^2} = \frac{-2\sum_{i=1}^{n} x_i}{\beta^3} + \frac{10n}{\beta^2} = \frac{-1}{\beta^2}\left(\frac{2\sum_{i=1}^{n} x_i}{\beta} - 10n\right)$

$\frac{\partial^2 l}{\partial \beta^2} = \frac{-1}{\left(\frac{\sum_{i=1}^{n} x_i}{10n}\right)^2}\left(\frac{2\sum_{i=1}^{n} x_i}{\frac{\sum_{i=1}^{n} x_i}{10n}} - 10n\right) = \frac{-(10n)^3}{\left(\sum_{i=1}^{n} x_i\right)^2} < 0$

[4]

iii)

The bias of $\hat{\beta}$ is given by Bias ($\hat{\beta}$) = E ($\hat{\beta}$) - β

E ($\hat{\beta}$) = $E\left\{\frac{\sum_{i=1}^{n} x_i}{10n}\right\} = \frac{nE(x_i)}{10n}$

As X is Gamma$\left(10, \frac{1}{\beta}\right)$; E (X) = 10 β

E ($\hat{\beta}$) = $\frac{nE(x_i)}{10n} = \frac{n(10\beta)}{10n} = \beta$

Hence bias ($\hat{\beta}$) = 0, so $\hat{\beta}$ is unbiased. [2]

iv)

From page 23 of the Tables, we have: $CRLB = -\frac{1}{E\left\{\frac{\partial^2 l}{\partial \beta^2}\right\}}$

Second log-differential: $\frac{\partial^2 l}{\partial \beta^2} = -2\frac{\sum_{i=1}^{n} x_i}{\beta^3} + \frac{10n}{\beta^2}$

$E\left[\frac{\partial^2 l}{\partial \beta^2}\right] = -2\frac{nE(x_i)}{\beta^3} + \frac{10n}{\beta^2} = -\frac{10n}{\beta^2}$

Therefore, CRLB = $\frac{\beta^2}{10n}$ [3]

v)

Variance of $\hat{\beta}$:

$V(\hat{\beta}) = V\left\{\frac{\sum_{i=1}^{n} x_i}{10n}\right\} = \frac{nV(x_i)}{100n^2}$

X is Gamma$\left(10, \frac{1}{\beta}\right)$; V (X) = 10 β^2

$V(\hat{\beta}) = \frac{nV(x_i)}{100n^2} = \frac{n(10\beta^2)}{100n^2} = \frac{\beta^2}{10n}$

We can see that $V(\hat{\beta}) = \frac{\beta^2}{10n}$ = CRLB obtained in (iii). [2]

vi)

From (i) above, MLE of β is:

$\hat{\beta} = \frac{\sum_{i=1}^{n} x_i}{10n}$

Using the invariance property of MLEs, the MLE of e^{β} is $e^{\hat{\beta}}$

$\hat{\beta} = \frac{\sum_{i=1}^{n} x_i}{10n} = \frac{(7+3+8+9+20)}{10(5)} = \frac{47}{50} = 0.94$

Therefore, MLE of $e^{\beta} = e^{0.94} = 2.56$ [2]

[14 Marks]

Solution 12:

i)

Let X be the number of hits and 'p' be the probability of a single missile hitting

$X \sim B(200, p) \approx N(200p, 200p(1-p))$

Or $\hat{p} \approx N(p, \frac{p(1-p)}{200})$

Estimating p in the variance term using $\hat{p} = \frac{144}{200}$ = 0.72, we have:

$z = \frac{144-200p}{\sqrt{40.32}}$ $or\ z = \frac{0.72-p}{\sqrt{0.001008}}$

95% confidence interval is P ($-1.96 \leq Z \leq 1.96$)

$$= P\left\{\frac{144 - 1.96\sqrt{40.32}}{200} < p < \frac{144 + 1.96\sqrt{40.32}}{200}\right\}$$

$$or\ P\left\{0.72 - 1.96\sqrt{0.001008} < p < 0.72 + 1.96\sqrt{0.001008}\right\}$$

Hence, our confidence interval for $\hat{p}$ is $0.72 \pm 0.062 = (0.658, 0.782)$ [3]

ii)

When a missile set is fired, the probability of hitting target is one minus probability of not hitting i.e. both missiles launched in a set failed = $(1-\mathrm{p})^2$
[0.5]
The probability δ that a target plane is hit is given by δ = $1-(1-\mathrm{p})^2$

From (i) above, 95% confidence interval for p is given by $(0.658, 0.782)$

0.95 = $P\,\{0.658 < p < 0.782\}$

0.95 = $P\left\{1 - (1-0.658)^2 < \delta < 1 - (1-0.782)^2\right\}$

So, the required confidence interval is (0.8829, 0.9526) [3]

[6 Marks]

Solution 13:

i)

Statistical test: Statistical / hypothesis testing begins with an assumption called a hypothesis that we make about a population parameter. A hypothesis is where we make a statement about something; for example the mean lifetime of smokers is less than that of non-smokers.

A hypothesis test is where we collect a representative sample and examine it to see if our hypothesis holds true.

Null Hypothesis: A hypothesis of no difference is called null hypothesis and is usually denoted by Ho. Null hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true.

It is very useful tool in test of significance. For example: If we want to find out whether the special classes for Students has benefited the students or not then Ho: special classes have not benefited the students.

Null hypothesis can sometimes be regarded as representing the current state of knowledge or belief about the value of the parameter being tested, the "status quo" hypothesis.

Alternative Hypothesis: Any hypothesis, which is complementary to the null hypothesis, is called an alternative hypothesis, usually denoted by H_1.

For example, if we want to test the null hypothesis that the population has a specified mean $\mu 0$ (say), i.e. Step 1: null hypothesis Ho: $\mu = \mu 0$ then Step 2.Alternative hypothesis may be H_1: $\mu <> \mu 0$ (i.e. $\mu > \mu 0$ or $\mu < \mu 0$) as a two tailed alternative.

[3]

ii)

Steps involved in hypothesis testing are:

- Specify the hypothesis to be tested
- Select a suitable statistical model
- Design and carry out an experiment study
- Calculate a test statistic
- Calculate the probability value
- Determine the conclusion of the test

[2]

iii)

a) Let p be the proportion of people for whom the drug induces sleep, then $X \sim B\ (20, p)$

We are interested in testing $H_0: p = 0.8$ against H_1: p < 0.8

We have to find the rejection region {x ≤ k} with $\alpha \approx 1\%$ => P (x ≤ k) = 0.01
Under $H_0: p = 0.8$ and $X \sim B\ (20, 0.8)$

From page 188 of Tables, we observe that for x = 11, n = 20 and p = 0.8
The probability P (x ≤ 11) = 0.0100. Hence, k = 11. [2.5]

b) Now, we are interested in testing $H_0: p = 0.8$ against H_1: p = 0.5

The probability of making type II error, β is the probability of accepting H_0 when it is false.

β = P (X > 11 when p = 0.5)= 1 - P (X≤ 11 when p = 0.5)

From page 188 of Tables, using probability for x = 11, n = 20 and p = 0.5
The probability P (x ≤ 11) = 0.7483. Hence, β = 0.2517 ≈ 25%. [2.5]

[10 Marks]

Solution 14:

i)

We know that $\hat{\beta} = \frac{S_{XY}}{S_{XX}} = \frac{54{,}243.00}{54{,}714.00} = 0.9914$

And $\hat{\alpha} = \bar{Y} - \hat{\beta}\ \bar{X} = 72.10 - 0.9914(72.00) = 0.7198$

Hence the prediction equation is $\hat{y} = 0.7198 + 0.9914x$ [2]

ii)

95% confidence interval for β is: $\hat{\beta} \pm t_{0.025,8}\sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}$

Where $\hat{\sigma}^2 = \frac{1}{n-2}\left\{S_{YY} - \frac{(S_{XY})^2}{S_{XX}}\right\} = \frac{1}{10-2}\left\{53{,}828.90 - \frac{(54{,}243.00)^2}{54{,}714.00}\right\} = 6.6057$

95% confidence interval for β is: $\hat{\beta} \pm 2.306\sqrt{\frac{6.6057}{54{,}714.00}}$

= (0.9661, 1.0167)

Alternate solution:

Where $\hat{\sigma}^2 = \frac{1}{n-2}\left\{S_{YY} - \frac{(S_{XY})^2}{S_{XX}}\right\} = \frac{1}{10-2}\left\{53{,}832.90 - \frac{(54{,}243.00)^2}{54{,}714.00}\right\} = 7.1057$

95% confidence interval for β is: $\hat{\beta} \pm 2.306\sqrt{\frac{7.1057}{54{,}714.00}}$

= (0.9651, 1.0177) [3]

iii)

For x = 100, the estimate of mean audited value is given by:

$\hat{\mu} = \hat{\alpha} + \hat{\beta}X_i = 0.7198 + 0.9914(100) = 99.8590$

Standard error of the estimate se $(\hat{\mu}) = \sqrt{\left\{\frac{1}{n} + \frac{(X_i - \bar{X})^2}{S_{XX}}\right\}\hat{\sigma}^2}$

Se $(\hat{\mu}) = \sqrt{\left\{\frac{1}{10} + \frac{(100-72.00)^2}{54{,}714.00}\right\}6.6057} = 0.8690$

95% confidence interval for $\hat{\mu}$ is $\hat{\mu} \pm t_{0.025,n-2}\, Se(\hat{\mu})$

Substituting values of $\hat{\mu} = 99.8590$; $t_{0.025,n-2} = 2.306$ and Se $(\hat{\mu}) = 0.8690$

We get 95% confidence limits for mean audited value = (97.855, 101.863)

Alternate solution:

Se $(\hat{\mu}) = \sqrt{\left\{\frac{1}{10} + \frac{(100-72.00)^2}{54{,}714.00}\right\}7.1057} = 0.9013$

95% confidence interval for $\hat{\mu}$ *is* $\hat{\mu} \pm t_{0.025,n-2}\, Se(\hat{\mu})$

Substituting values of $\hat{\mu} = 99.8590$; $t_{0.025,n-2} = 2.306$ *and Se* $(\hat{\mu})$ *= 0.9013*

We get 95% confidence limits for mean audited value = (97.781, 101.937)

[3]

iv)

95% confidence interval for $\hat{y}$ is $\hat{y} \pm t_{0.025,n-2}$ Se $(\hat{y})$; where standard error of the individual audited value estimate, se $(\hat{y})$ = $\sqrt{\left\{1+\frac{1}{n}+\frac{(X_i-\bar{X})^2}{S_{XX}}\right\}\hat{\sigma}^2}$

For any confidence interval, the minimum length is attained at minimum se $(\hat{y})$.

By observing the expression for se $(\hat{y})$, minimum se $(\hat{y})$ = $\sqrt{\left\{1+\frac{1}{n}\right\}\hat{\sigma}^2}$

Se $(\hat{y})$ will be minimum at $X_i = \bar{X}$ since $X_i$term is squared and minimum can be zero at $X_i = \bar{X}$ = 72 [2]

Alternatively, to find the minimum se $(\hat{y})$, we can differentiate se $(\hat{y})$ with respect to $X_i$and solve by setting it to zero to get $X_i = \bar{X}$.

v)

Coefficient of Determination $R^2 = \frac{(S_{XY})^2}{S_{XX}S_{YY}} = \frac{(54{,}243.00)^2}{(54{,}714.00)\ (53{,}828.90)}$ = 99.90%

Alternate solution:

Coefficient of Determination $R^2 = \frac{(S_{XY})^2}{S_{XX}S_{YY}} = \frac{(54{,}243.00)^2}{(54{,}714.00)\ (53{,}832.90)}$ = 99.89%

Coefficient of determination is used to measure the goodness of fit of a linear regression model. A value of 99.89% means the model is a good fit. [2]

[12 Marks]

Solution 15:

i)

Ho: Each surveyor has the same average of estimated prices

H_1: There are differences among the average of estimated prices by different surveyors.

For the given data, summary measures are:

$y_{1.}$= 37.50; $y_{2.}$= 38.00; $y_{3.} = 37.00$; $y_{4.} = 36.00$; $y_{..}$ = 148.50; $\sum\sum y_{ij}^2$= 1107.71

SS T = 1107.71 – $148.50^2/20$ = 5.0975

SS B = $(37.50^2/5 + 38.00^2/5 + 37.00^2/5 + 36.00^2/5) - 148.50^2/12$ = 0.4375

SS R = 5.0975 – 0.4375 = 4.6600

Source of variation	Degrees of Freedom	Sum of squares	Mean squares
Between treatments	3	0.4375	0.1458
Residuals	16	4.6600	0.2913
Total	19	5.0975	

The variance ratio is F = 0.1458 / 0.2913 = 0.5007

Under Ho, this has an F (3, 16) distribution. The 10% critical point is 2.462, so we cannot reject Ho, and we conclude that the average of estimated prices don't differ between the four surveyors. [6]

ii)

Sample means: $\bar{y}_{2.}$ = 7.60, $\bar{y}_{4.}$ = 7.20

For the second and fourth surveyors, least significant difference at 5% level is

$$t_{(0.025,n-k)}\ \hat{\sigma}\left(\frac{1}{\mathrm{n}_2}+\frac{1}{\mathrm{n}_4}\right)^{0.5}$$

$$= (2.120)\ (0.2913)^{0.5}\left(\frac{1}{5}+\frac{1}{5}\right)^{0.5} = 0.7236$$

The least significant difference of 0.7236 is more than the difference of 0.40 (= 7.60 – 7.20) between the average of estimated prices by the two surveyors. Hence, the average of estimated prices by surveyor 2 is not significantly greater than that of surveyor 4. [3]

[9 Marks]

****