# INSTITUTE OF ACTUARIES OF INDIA 

## EXAMINATIONS

## $05^{\text {th }}$ May 2016

## Subject CT8 - Financial Economics

Time allowed: Three Hours ( 10.30 - 13.30 Hrs.)
Total Marks: 100

## INSTRUCTIONS TO THE CANDIDATES

1. Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.
2. Mark allocations are shown in brackets.
3. Attempt all questions, beginning your answer to each question on a separate sheet.
4. Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.

## AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.
Q. 1) Comment on which form of market efficiency is supported/contradicted by the following scenarios:
i) Market over reacts to the news and corrects itself over couple of trading sessions.
ii) An investor earns returns that are significantly higher than those predicted by a log-normal model fitted for a particular index using technical analysis.
iii) Equity yields are correlated with the yields implied in the last three trading sessions and show an element of mean revision over a 4-5 year timeframe.
Q. 2) i) A stock price is currently Rs. 400 . Assume that the expected return from the stock is $17 \%$ and that its volatility is $30 \%$. Derive and determine the probability distribution for the rate of return (with continuous compounding) earned over a period of two years? State your assumptions.
ii) Derive the differential equation of stock price $S$ on the basis of your assumption in part 2(i).
iii) Suppose that F is a function of a stock price S . Let $\sigma_{S}$ and $\sigma_{F}$ are the volatilities of S and F respectively. Show that when the growth rate on S increases by $\pi \sigma_{S}$, the growth rate of F increases by $\pi \sigma_{F}$, where $\pi$ is a constant
Q. 3) There are two equity stocks with correlation coefficient of ' -0.6 ' listed in a market with the share price, mean return and variance as shown in the following table. Short-selling is not permitted in the market and one cannot have infinitely divisible holdings:

|  | Share Price (Rs.) | Returns | Volatility |
| :---: | :---: | :---: | :---: |
| Share A | 100 | $10 \%$ | $10 \%$ |
| Share B | 200 | $12 \%$ | $25 \%$ |

An investor wants to invest Rs. 10000 for one year in the market and approached you for advice.
i) Calculate the proportion of funds that you will advise the investor to invest in Share A if the variance on the portfolio is to be minimized. Hence calculate the expected return on the portfolio. (Hint: $x_{A}=\frac{V_{A}-C_{A B}}{V_{A}+V_{B}-2 C_{A B}}$ )
ii) How would your answer change if the correlation coefficient is +0.6
iii) Comment on risk appetite of the investor for whom the above advice would be appropriate.
Q. 4) A financial advisory company is conducting a survey to check applicability of behavioural finance theories to a particular population and has devised the following questionnaire. According to you which theme of behavioural finance is the advisory trying to survey by asking each of the following questions?
i) Would a prior investment decision that resulted in a loss stop you from making a similar decision, even if the new investment appears to be the best alternative?
ii) How frequently do you review your investment portfolio?
iii) A popular analyst has commented that markets are expected to provide returns in excess of $15 \%$ in the coming year. Will you invest in equities with a hope to earn atleast $15 \%$ for next year?
iv) Would you prefer to manage your portfolio yourself or rely on views of expert analysts?
Q. 5) A portfolio manager has recently announced a derivative which would pay off $S_{T}^{K}$ at time T where $\mathrm{S}_{\mathrm{T}}$ is the stock price at time T and K is constant. It is known to the investors that stock price follows Geometric Brownian motion with drift $\mu$ and volatility $\sigma$ and the price of the derivative at time t is $f(t, T) S_{t}^{K}$
i) Using Black Schole's PDE, derive a differential equation satisfied by $\mathrm{f}(\mathrm{t}, \mathrm{T})$.
ii) Determine boundary condition satisfied by $\mathrm{f}(\mathrm{t}, \mathrm{T})$.
iii) Show that $f(t, T)=\exp \left(\left(-r(K-1)-\frac{1}{2} \sigma^{2} K(K-1)\right)(T-t)\right)$ where r is the risk free rate of interest rate.
iv) Show that the process is a martingale if $\mu=\mathrm{r}(\mathrm{K}-1) / \mathrm{K}$
Q. 6) An individual has disclosed the level of premiums he is prepared to pay to secure against the following events:
i) Property insurance $=$ Rs. 300,000 against total loss of property and contents including cash amounting to Rs. $10,00,000$. (Probability of event $=25 \%$ )
ii) Property insurance $=$ Rs. 150,000 against total loss of property but without contents amounting to Rs. 500,000 (probability of event $=15 \%$ )

The individual has also stated that he would prefer to take a certain Rs. 500,000 than take a gamble which pays Rs. $1,000,000$ with $60 \%$ probability.

Sketch a graph of investor's utility function for the region (Rs. 500,000 to Rs. 1,000,000)
Q. 7) i) The price of a stock currently stands at Rs. 250. The stock price is expected to be either Rs. 400 or Rs. 200 after a 3 months period. The risk free rate of interest with quarterly compounding is $8 \%$ per annum. Given that the strike price is Rs. 250 , calculate the value of the three month European put option.
ii) Verify that the no-arbitrage argument and risk neutral valuation gives the same answer.
Q. 8) A portfolio manager is managing a well-diversified portfolio of Rs. 50cr which closely follows the BSE Sensex. He has been given a mandate to hedge the portfolio from a potential reduction in value by more than $10 \%$ in the next 6 months to come.

The current value of the BSE Sensex is 25,000 , risk free rate is $6.0 \%$ per annum and dividend yield on the portfolio he is managing is $4 \%$ per annum whereas that of the Sensex is $3 \%$ per annum, volatility of the index is $30 \%$ and Beta of the portfolio is 1.5
i) How much cost would the portfolio manager incur if he decides to hedge the portfolio by buying appropriate European options?
ii) Suggest and quantify an alternative strategy for the portfolio manager resulting into the same hedge
iii) Determine the delta of the portfolio and the amount of risk free securities he needs to hold to delta hedge the portfolio
Q. 9) List the desirable characteristics of term structure model and confirm whether these are met by Cox-Ingersoll-Ross model.
Q. 10) Currently a company has the following position:

Asset

$$
\text { = Rs. } 100 \text { crore }
$$

$$
\text { Volatility of its assets } \quad=25.00 \%
$$

$$
\text { Risk free rate } \quad=8.00 \%
$$

Company is in need of fresh capital for its projects and wishes to raise the same by issuing zero coupon bonds that will mature after 5 years. The amount to be paid on redemption would be Rs. 90 crore.
i) Under Merton Model, determine the probability of default on the debt raised by the company clearly stating the assumptions made.
ii) Determine the annualized yield on the zero coupon bonds.
iii) Determine the recovery ratio based on 2 state model assuming probability of default same as that calculated in (i) but the company wants to maintain the credit spread at 25 basis points.
iv) Describe different types of models that can be used to model Credit Risk.
Q. 11) i) State the difference between Capital Market Line and Security Market Line
ii) Show that a portfolio p which lies on security market line (SML) is not an efficient combination of the riskless asset and the market portfolio if $\rho\left(\mathrm{r}_{\mathrm{p}}, \mathrm{r}_{\mathrm{m}}\right)<1$ where $\rho$ is the correlation coefficient of the portfolio and the market returns $r_{p}$ and $r_{m}$ respectively.

