# INSTITUTE OF ACTUARIES OF INDIA

# EXAMINATIONS

## 27th April 2016

## Subject CT6 – Statistical Methods

## Time allowed: Three Hours (10.30 – 13.30 Hrs.)

## Total Marks: 100

### *INSTRUCTIONS TO THE CANDIDATES*

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***

4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

| **AT THE END OF THE EXAMINATION** |
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| **Please return your answer book and this question paper to the supervisor separately.** |

**Q. 1)** $Y_t$ follows the below process:

$$Y_t^2 - \beta_1 e_t^2 Y_{t-1}^2 = 2(Y_t - \beta_1 e_t^2 Y_{t-1})\mu - (1-\beta_1 e_t^2)\mu^2 + \beta_0 e_t^2$$

Where $e_t$ are independent (from $Y_t$, $Y_{t-1}$,…) standard normal random variable.

**i)** Calculate Cov ($Y_t$, $Y_{t-s}$). (6)

**ii)** Find out whether $Y_t$ and $Y_{t-s}$ are dependent. (2)

Now $X_t$ follows below process:

$$X_t = 0.5Y_t + 0.3t + 0.1$$

**iii)** Find out whether first order difference of $X_t$ is stationary. (5)
**[13]**

**Q. 2)** The number of claims in an insurance company follows type 2 negative binomial distribution with mean and variance equal to 100 and 150 respectively. Individual claim amounts follow exponential distribution with mean 100.

**i)** What are the advantages of negative binomial distribution compared to Poisson distribution for number of claims? (1)

**ii)** Deduce MGF of aggregate claims and calculate mean and variance. (7)

In a different insurance company the individual claim amounts follow the same distribution with parameter μ. However the number of claims follows binomial distribution with mean 40 and variance 16.

**iii)** If the aggregate claims distribution is same for both the companies then find out μ. (5)
**[13]**

**Q. 3)** The non - cumulative claims for a portfolio of insurance policies is given below (In Rs 5000):

| **Accident year** | **Development year** | | | |
|---|---|---|---|---|
| | **0** | **1** | **2** | **3** |
| **2011** | 1572 | 820 | 425 | 325 |
| **2012** | 1600 | 750 | 450 | |
| **2013** | 1823 | 900 | | |
| **2014** | 1700 | | | |

**i)** The earned premium for the Accident years 2011,2012,2013,2014 are 17500000, 19250000, 1850000 and 2050000 respectively

Calculate the emerging liability using the BF method. (5)

**ii)** However an actuarial student working for the company wishes to take into account the effect of inflation in the mathematical reserve calculation. He assumes that the future inflation is 10 % p.a and past inflation is based on the following index(measured to the midpoint of the relevant year):

| **Year** | **Index** |
|---|---|
| 2011 | 400 |
| 2012 | 429 |
| 2013 | 465 |
| 2014 | 516 |

Calculate the revised estimate of the liability using inflation adjusted chain ladder method using development factors. (9)

Note: Claims are assumed to be fully run off – at the end of development year 3. **[14]**

**Q. 4)** A newly registered insurance company ABC writes three classes of insurance business A, B and C.

**i)** Claims on classes A and B are independent and follow the Poisson process.

The mean individual claim amount on class A is exponentially distributed and equals 3000 and the expected number of claims per year are 20, while for class B, individual claim amounts are fixed at Rs 2000 with the expected number of claims being 10.

Let S denote the aggregate claim arising for ABC in a year from class A and B

**a)** Calculate the mean of S (2)

**b)** Calculate the variance of S (2)

The insurer uses a loading of 25 % in its premium calculation.

**c)** Find out the initial capital required to ensure that the probability of ruin at the end of first year is 0.025. (3)

**ii)** Also ABC is planning to buy reinsurance for class C on which a maximum of one claim is possible in any year. The possible claim amounts on an individual policy can take any one of the following values Rs 2000,3000,4000,5000.

The reinsurer offers the following different reinsurance arrangements for the insurer to choose

Type 1: proportional reinsurance of 25% with premium of 600

Type 2: excess of loss with retention level 4000 and premium of 300

**a)** If the annual premium under the insurance contract is Rs 1500 for Class C, find out the loss table for the insurer under the two reinsurance arrangements and compare this with the possibility when reinsurance is not opted for. (6)

**b)** Find out the minimax solution to this problem by comparing all three scenarios (2)

**[15]**

**Q. 5)** At the end of last year, a new laptop manufacturer approached an insurance company for providing insurance for laptops sold during the first year of its operation.

Based on existing insurance contracts in its portfolio, the insurer estimated the probability of failure over the coming year for each laptop to be "p". After a year the insurer observes that 92 laptops suffer from complete failure during the year of insurance out of 9000 laptops insured.

Assuming that the prior distribution of p is beta with mean 0.013 and standard deviation 0.004 find out the posterior distribution of p. **[6]**

**Q. 6)** The claims from Motor (X1) and Fire (X2) Insurance contracts follow normal distributions with parameters depend on θ and they are conditionally independent given θ. θ follows normal distribution with mean 5 and variance 10.

$$E(X1|\theta)=10+\theta, E(X2|\theta)=5+3\theta, Var(X1|\theta)=2+3\theta^2, Var(X2|\theta)=3+5\theta^2$$

**i)** Find the unconditional mean and variance of X1 and X2. (5)

**ii)** Find unconditional mean of X1X2 and check whether X1 and X2 are unconditionally independent. (3)
**[8]**

**Q. 7)** **i)** In a Bakery shop, the cost of producing breads is x per unit and selling price is y per unit. Each day after 9 PM, the owner reduces the price of any unsold items to z per unit (z < y) and finishes his stock for the day. The demand for the bread is a random variable D with probability distribution p(d). Prove that the optimum number of units (t) to produce any day follows the below:

$$\Pr(D<t)<\frac{y-x}{y-z}$$ (8)

Now, x = 35, y = 70, z = 25 and probability distribution of demand is given in the below table:

| Quantity (t) | Pr (D < t) |
|---|---|
| 100 | 0.0 |
| 200 | 0.1 |
| 300 | 0.4 |
| 400 | 0.6 |
| 500 | 0.75 |
| 600 | 0.8 |
| 700 | 0.85 |
| 800 | 0.9 |
| 900 | 0.95 |
| 1000 | 1.0 |

**ii)** Calculate optimum quantity the bakery should produce each day. (1)

**iii)** Calculate the expected profit if he chooses to produce the optimum quantity. (2)
**[11]**

**Q. 8)** A small health insurer has taken out individual excess of loss reinsurance on a group of health insurance policies, in which the cost of major surgery is reimbursed. The retention limit is INR 10,00,000. The gross claims (in INR thousands) made to the direct health insurer have a distribution with density function:

$f(x) = 2cx \exp(-cx^2)$ , $x > 0$

The insurer finds than over a period of three months, the insurer paid out the following amounts in claims (in INR):

650000, 905000,486000, 345000, 609000, 290000, 689000, 388000.

In addition to this there were 2 claims above the retention limit and hence involved the reinsurer.

Calculate the maximum likelihood estimate of *c*. **[7]**

**Q. 9)** **i)** Write down the key components of Generalised Linear Model (GLM). (1)

**ii)** Define different types of covariate. (1)

**iii)** An employer provides a fixed pension to its employees who retire from the company at exact age 60 after serving the company for at least 15 years. The pension depends upon the last drawn salary only and continues till the employee survives. A mortality study of the pensioners was carried out where the total duration of pension payment of the 249 ex-pensioners (who died as pensioner) were recorded.

Suppose that $Y_i$ represents the total duration of pension payments (in years) of the *ith* pensioner and $x_i$ represent the logarithm (to the base 10) of the *i*th pensioner's last drawn annual salary in INR lacs (for i = 1,2,3,....,249).

The response variables *Yi* are assumed to be exponentially distributed. A possible specification for $E(Y_i)$ is $E(Y_i) = \exp(a + bx_i)$ to ensure $E(Y_i)$ is greater or equal to zero for all values of $x_i$ .

**a)** Write down the natural link function associated with the linear predictor

$\eta_i = a + bx_i$. (2)

**b)** Use this link function and linear predictor to derive the equations that must be solved in order to obtain the maximum likelihood estimates of a and b. (4)

**c)** The maximum likelihood estimate of a is derived from the available data by the company as 8.477 and the standard error of estimation of a is calculated as 1.655.

Find an approximate 95% confidence interval for a and interpret this result. (2)

**d)** The following two models are suggested by a consultant:

Model A: $E(Yi) = a$

Model B: $E(Yi) = a + bx_i$

The deviance for Model A is found to be 226.282 and the deviance for Model B is 219.457. Test the null hypothesis that $b = 0$ against the alternative hypothesis that $b \neq 0$ stating your conclusion clearly. (3)

**[13]**

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