INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

26th April 2016

Subject CT3 – Probability & Mathematical Statistics

Time allowed: Three Hours (10.30 – 13.30 Hrs.)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.***
4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) The following table gives the frequency distribution of the number of car insurance claims based on a sample of 100 policies during last year.

Number of claims (*x*)	1	2	3	4	5
Number of policies (*f*)	5	30	45	10	10

Calculate the mean, median, mode, range and variance of number of claims. **[5]**

Q. 2) Given that $P(A) = 0.3$, $P(B) = 0.6$ and $P(A|B) = 0.4$; Find $P(A^c \cap B^c)$. **[3]**

Q. 3) Suppose that:

- 30% of adults in India are obese;
- among the obese adults, 2 out of 5 have hypertension; and
- among the non-obese adults, 1 out of 10 have hypertension

i) Find the probability that a randomly selected adult is:

a) Obese with hypertension. (1)

b) Obese without hypertension. (1)

c) Non-Obese with hypertension. (1)

d) Non-Obese but without hypertension. (1)

e) With hypertension. (1)

ii) Assume that the above probabilities are applicable for all the members of a club. The Club has decided to sponsor the yearly cost of medicines for its hypertensed members. The cost is Rs. 4000 for an obese adult and Rs. 2000 for a non-obese adult. If the club has 1000 members, calculate the yearly expected cost of the medicines incurred by the club. (Assume that the health status of members remain same and no one leaves or joins the club during the year) (3)

[8]

Q. 4) Two samples *A* and *B* of unequal sample size are taken from a population. The mean and variance are given in the table below for each of the two samples. The mean of the combined sample is also given.

	Sample size	***Sample mean***	***Sample variance***
A	*Unknown*	*10*	*30*
B	*Unknown*	*18*	*72*
Combined	*40*	*16*	*?*

i) Find the sizes of the samples. (2)

ii) Find the variance of the combined sample. (4)

iii) Comment on answer in (ii) above by comparing with variances of the original samples. (1)

[7]

Q. 5) Let $X_1, X_2, \dots, X_n$ be *iid* random variables from exponential distribution with parameter λ. Find the *pdf* of $Max(X_1, X_2, \dots, X_n)$. **[4]**

Q. 6) Let *X* denote the time taken by a worker to complete a specified work in a factory. For a given worker, the distribution of *X* is modelled as an exponential distribution with unknown mean *u* that varies across the work force. *U* is treated as a Uniform random variable over *(a, b)*, i.e.

$X \mid (U = u) \sim Exponential\ (u)$
$U \sim Uniform\ (a, b)$

Find the mean and variance of the marginal distribution of *X*. **[6]**

Q. 7) A company recruits 100 new employees. A newly recruited employee is eligible for pension benefits if he/she continues to be in the organization for at least 5 years. The probability of an employee to continue for at least 5 years is 0.20.

Calculate an approximate value for probability that more than 25 of newly recruited employees will be eligible to receive the pension benefits. **[4]**

Q. 8) It is known from past experience that the daily tip amount, a waiter in a restaurant gets, is a random variable with mean Rs.100 and standard deviation Rs. 10.

i) Assuming that the number of tips is sufficiently large, calculate the number of tips required to ensure with at least 0.95 probability that the average daily tip would exceed Rs. 98. (3)

ii) Given that the waiter gets 64 tips on a particular day and the tip amounts are independent. What is the probability that the total tips amount is greater than Rs.6500 on the given day? (3)

[6]

Q. 9) The following are the summary measures of birth weights (in grams) of babies in a city

Gender	***Mean***	***Standard Deviation***
Male	*3000*	*300*
Female	*3500*	*400*

Assuming that the birth weights are independently normally distributed for the two genders, calculate the probability that at birth a boy outweighs a girl. **[3]**

Q. 10) Anti-collision device is used to transmit a signal in case of the collision of two trains. The manufacturing company *A* makes 80% devices, company *B* makes 15% devices and the rest is made by company *C*. The device made by company *A* has 4% rate of defects, device made by company *B* has 6% rate of defects and the device made by company *C* has 9% rate of defects.

If a randomly selected anti - collision device is tested and is found to be defective, find the probability that it was made by the company *A*. **[3]**

Q. 11) The probability density function of a random variable *X*, is given by

$$f(x) = \frac{x^9 e^{\left(\frac{-x}{\beta}\right)}}{\beta^{10} 9!} \; ; x > 0, \beta > 0$$

Let $X_1, X_2, \ldots, X_n$ denote a random sample of size *n* on X.

i) Identify the distribution of *X* (1)

ii) Obtain the MLE of β. (4)

iii) Show that the MLE of β is unbiased. (2)

iv) Obtain the CRLB for the MLE of β. (3)

v) Show that the MLE of β has variance equal to the CRLB in (iv). (2)

vi) Suppose that the sample values are 7,3,8,9 and 20. Obtain the ML estimate of e^{β} (2)
[14]

Q. 12) The Defence Research and Development Corporation is testing the effectiveness of a new land to air missile which is believed to be more effective than current defence systems at hitting its target under poor lighting conditions. The corporation has carried out a series of 200 trials in which the missile was required to respond to an enemy fighter plane following a randomly selected trajectory. The missile successfully hits its target in 144 of these trials.

i) Obtain the 95% confidence interval for the probability that one of the new missiles will hit its target (3)

ii) The new missiles are to be deployed in sets of two, which will be launched together, separated by three seconds interval, at the same target. Find a 95% confidence interval for the probability that a target plane will be hit when a missile set is fired. (3)
[6]

Q. 13) **i)** In the context of hypothesis testing, define a statistical test, null hypothesis and alternate hypothesis. (3)

ii) List the steps involved in hypothesis testing. (2)

iii) An experimenter has prepared a drug dosage level that she claims will induce sleep for 80% of people suffering from insomnia. After examining the dosage, we feel that her claims regarding the effectiveness of the dosage are inflated. In an attempt to disprove her claim, we administer her prescribed dosage to 20 insomniacs and we observe *X*, the number for whom the drug dose induces sleep. Assume that the rejection region $\{x \leq k\}$ is used.

a) Find the value of *k* so that *P (Type I error)*, α , is approximately at 1% level (2.5)

b) For the rejection region in part (iii), find *P (Type II error)*, β , when the proportion of people suffering from insomnia is 1/2. (2.5)

[10]

Q. 14) Auditors are often required to compare the audited (or current) value of an inventory item with the book (or listed) value. If a company is keeping its inventory and books up to date, there should be a strong linear relationship between the audited and book values: An Accountant intends to fit a linear regression model. He sampled ten inventory items and obtained the audited and book values shown in the following table.

Book - X	10	12	9	27	47	112	36	241	59	167
Audited-Y	9	14	7	29	45	109	40	238	60	170

$\bar{X}$ =72.00; $\bar{Y}$= 72.10; S_{XX}=54,714.00; S_{YY}=53,828.90 and S_Y=54,243.00

i) Fit a Linear regression model: $Y = \alpha + \beta X + e$ for the above data. (2)

ii) Obtain a 95% confidence interval for β, the slope parameter. (3)

iii) If the book value x = 100, find a 95% confidence interval for the predicted mean audited value $\hat{\mu} = E[Y|X = x]$. (3)

iv) Find the book value x for which the 95% confidence interval for $\hat{y}$, the predicted individual audited value, has minimum length. (2)

v) Calculate coefficient of determination and interpret. (2)

[12]

Q. 15) A real estate company has employed four surveyors to value five similar properties. The five properties that are similar should have same values and may differ slightly depending on location and age of the property. Any surveyor's estimated price of a property is assumed to be normally distributed with mean equal to average value (in his/her eyes) of similar properties and same variance for all surveyors' and all properties. The employer feels that there may be differences in mean of estimated prices by the four surveyors. The estimated prices in millions of each property by each surveyor are recorded and is summarised below.

Surveyor	*Sum of Prices*	*Sum of squares*
1	37.50	282.79
2	38.00	289.50
3	37.00	274.74
4	36.00	260.68

i) Show that there is no difference at the 10% level in the mean of estimated prices by the four surveyors. (6)

ii) The employer has a view that the mean of estimated prices of surveyor 4 is significantly less than that of surveyor 2. Comment on the employer's view using least significant difference approach. (3)

[9]

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