INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

30th November 2012

Subject ST6 – Finance and Investment B

Time allowed: Three Hours (9.45* – 13.00)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*

2. ** You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You have then three hours to complete the paper.*

3. *You must not start writing your answers in the answer sheet unless instructed to do so by the supervisor.*

4. *The answers are not expected to be any country or jurisdication specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*

5. *Attempt all questions, beginning your answer to each question on a separate sheet.*

6. *Mark allocations are shown in brackets.*

7. *Please check if you have received complete Question paper and no page is missing. If so, kindly get a new set of Question paper from the Invigilator.*

AT THE END OF THE EXAMINATION
Please return your answer book and this question paper to the supervisor separately.

Q.1) [i] Consider the process,
$X_t = B_{2t} - B_t, where\ B_t is\ a\ standard\ brownian\ motion, 0 \leq t < \infty$
Is it a Gaussian process? Can you find the mean and variance? Is it Brownian motion? [3]

[ii] Let X_t and Y_t be independent Brownian motions. Let $Z_t = (X_t + Y_t)/\sqrt{2}$. Is Z_t a Gaussian process? Is it a Brownian motion? [3]

[iii] Consider two non-dividend paying stocks X_t and Y_t. There is a single source of uncertainty which is captured by a standard Brownian motion $\{B_t\ ,\ t \geq 0\ \}$. The prices of the assets satisfy the stochastic differential equations:

$$\frac{dX}{X} = 0.08dt + 0.15\ dB$$

$$\frac{dY}{Y} = \alpha dt + 0.08\ dB$$

The continuously compounded risk free interest rate is 0.06. Determine α. [3]

[9]

Q.2) [i] An at the money American call option with a term of 6 month is to be written on a dividend paying stock where the ex-dividend dates are 2 months and 5 months away from now. The dividend on each ex-dividend date is expected to be Rs 0.5. The current share price is Rs 40 and the risk-free rate is 9% per annum compounded continuously. Calculate the price of the option using Black's approximation. Explain whether the answer given by Black's approach understate or overstate the true option value. [7]

[ii] It was decided to use delta hedge to manage this option. Draw the graph of the likely movement of the delta with the passage of time for at the money, in the money and out of money call option. [2]

[iii] A mutual fund announces that the salaries of its fund managers will depend on the performance of the fund. If the fund loses money, the salaries would be zero and if it makes a profit then the salaries will be proportional to the profit. Describe the salary as a derivative. How is a fund manager motivated to behave with this type of remuneration package? [3]

[12]

Q.3) [i] An American call option on the futures contract is traded along with an American call option on the underlying currency USD where all have the same maturity. Please argue whether the prices of both the options will be equal or different. [3]

[ii] The exchange rate currently on USD is INR 55. There is expected to be sovereign rating announcement tomorrow which may lead the exchange rate either to 48 or to 62. What are the problems in using Black Scholes to value one month European call option on the exchange rate of USD? [3]

[6]

Q.4) [i] Let S_t represent the stock price at time t for a non-dividend paying stock. Show that the price of the call option with exercise price of K at time T can be written as follows:
$c = e^{-rT} \int_{S_T=K}^{\infty} (S_T - K)\ f(S_T)\ dS_T$ where r is the constant risk free interest rate. [2]

[ii] Show that $f(K) = e^{rT} \frac{\partial^2 c}{\partial K^2}$ [3]

[iii] Explain how you can estimate the risk neutral probability distribution from observed prices on traded option. [3]

[8]

Q.5) Consider a two period binomial model with t=0,1,2. There are a stock and a risk free asset. The initial stock price is 4 and the stock price doubles with probability 2/3 drops to half with probability 1/3 each period. The risk free rate is 25% per period compounded per period.

[i] Compute the risk neutral probability at each node [2]

[ii] Compute the discrete world equivalent of Radon Nikodym derivative of the risk neutral measure with respect to the physical measure at each node [2]

[iii] Price an european lookback option with payoff at t=2 equal to $Max\{0,(max_{0\le t\le 2} S_t)-S_2\}$ using risk-neutral probability [3]

[iv] Calculate the above price using replicating portfolio method and show that it is same as above. [4]

[v] Use European call as control variate to calculate a better approximation of the above calculated price. [3]

[vi] Estimate delta and theta for the above derivative price at t=0. [2]

[16]

Q.6) A bond dealer provides the following selected information on a portfolio of fixed income securities

Par Value(lakh)	Market Price	Coupon	Modified Duration	Effective duration	effective Convextity
500	100	6.75%	8.5	8.5	145
250	94.3	5.25%	6	1.75	45
300	93	7%	8.5	8.5	125
100	105	8%	9	4.5	-65

[i] What is the effective duration for the portfolio and calculate change in value of the portfolio for a basis point change in the yield? [3]

[ii] Which of the bond(s) most likely to has(have) no embedded options and why? [2]

[iii] Which bonds(s) is (are) likely callable and puttable and why? [3]

[iv] What is the approximate price change for the 7% bond if its yield to maturity increases by 25 basis points? [2]

[v] Why might portfolio effective duration be an inadequate measure of interest rate risk for a bond portfolio even if we assume the bond effective duration are correct? [2]

[12]

Q.7) An Insurance company has net equity exposure of 100 Cr, asset liability management team has suggested change in asset allocation as specified in the table

	Current	Future
Indian equity	50%	80%
UK equity	25%	10%
US equity	25%	10%

The company has decided to use stock future for implementing this transaction and aim to finish the stock allocation in 3 months.

[i] State advantage of using stock futures compared to direct sale of asset in the market. [3]

[ii] What stock futures should company buy and sell to attain the required asset allocation? Calculate value of fixed leg of each futures transaction where spot USD/INR – 55 and GBP/INR – 88. Use the following information:

- Futures on UK equities are index linked with FTSE 100 underlying priced 5450 GBP as of today(transaction date)
- Futures on US equities are index linked with Dow Jones underlying priced 13600 USD as of today(transaction date)
- Futures on Indian equities are index linked with BSE underlying priced 15000 RS as of today(transaction date)
- 3 month interest rate India – 3% UK/US =1% [4]

[iii] Design a hedge to be implemented at the start of future contract to limit the losses due to currency movement. Also explain how will you account for interest rate differential in different economies. [4]

[iv] The market after 3 months is as follows:

Indian interest rate	5%
UK interest rate	10%
US interest rate	5%
USD/INR spot rate	55
GBP/INR spot rate	90

Calculate net profit and loss of the company in following scenarios

[a] No equity futures (assume same number of indexes are sold/bought as future contracts on date of exchange)

[b] Equity futures with NO currency hedge

[c] Equity futures with currency hedge [8]

[19]

Q.8) [i] Define cap. How many options would you have in 2 year cap resetting quarterly? [2]

[ii] Suppose you have purchased a 2 year cap annual reset and a strike rate of 5.0% on a notional principal of Rs 25 Cr. Assume current interest rate to be 6% and each year it can either go up 1.2 times or go down 0.8 times. Use binomial model to calculate price of all the options in this cap and price of cap itself. (1.2 times of 6% means 7.2%) [8]

[10]

Q.9) [i] How would you price a bond with maturity T in arbitrage free market and risk neutral valuation. (assume payment of d if default or 1 if not). Make simplified assumptions to write this price in terms of risk free rate and default probability p. [4]

[ii] Suppose probability of default before time T for this bond is given by random variable $\varphi(-Z)$ where φ represents the cumulative probability distribution for a standard normal and Z is a standard normal random variable. Assume term of the bond to be 5 years, risk free rate of 6% (pa compounded continuously) and recovery rate to be 40%. Use monte carlo method (5 simulations) to calculate the credit spread over risk free rate for this corporate bond. The following are five random numbers from uniform (0,1) distribution (0.985,0.707,0.447,0.051,0.783). [4]

[8]

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