INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

29th November 2012

Subject CT8 – Financial Economics

Time allowed: Three Hours (10.00 – 13.00)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. **Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception**
2. **Mark allocations are shown in brackets.**
3. **Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.**
4. **Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.**

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q.1) Consider an asset generating income at q% per annum with continuous compounding.

[i] Derive the forward price, K, of a contract issued at time 0 maturing at time T to purchase one unit of the asset. [5]

Wipro Limited enters into a forward contract with SBI to sell one USD for INR 50 after 3 months.

Three months later, the spot exchange rate is $USD\ 1 = INR\ 53$.

Wipro Limited asks SBI to roll over the contract for 6 more months.

The prevailing risk-free interest rates with continuous compounding for all maturities in India and the US are 8% per annum and 5% per annum respectively.

[ii] Calculate the delivery price of the rolled over contract such that no money is required to change hands immediately. Describe your calculations. [5]

[10]

Q.2) The current price of Wipro stock is Rs. 300 per share. Over each of the next two 3-month periods it is expected to go up by 8% or decline by 10%. The risk-free rate of interest is 6% per annum with continuous compounding.

[i] Construct a binomial tree and calculate the value of a 6-month European call option with a strike price of Rs. 310 using a two-step binomial model? [4]

[ii] Calculate the price of a 6-month American put option on the stock with the same strike price. [3]

[7]

Q.3) The *Garman-Kohlhagen* formula for price 'c' at time 0 of a European call option on a non-dividend paying stock is given as:

$$c = S_0\Phi(d_1) - Ke^{-rT}\Phi(d_2)$$

[i] Define the terms used in the above formula. [2]

[ii] Show that $S_0\phi(d_1) = Ke^{-rT}\phi(d_2)$, where $\phi(x)$ is the probability distribution function of a standard normal variable. [4]

[iii] Show that the delta of call option is $\Phi(d_1)$. [4]

[10]

Q.4) **[i]** Define the continuous-time lognormal model of security prices. [2]

[ii] Describe briefly any three reasons why continuous-time lognormal model may be inappropriate for modeling investment returns. [3]

[iii] Describe a *credit event* in respect of a bond issued by a corporate entity. [2]

[7]

Q.5) **[i]** Describe the impact on volatility of short-rate, *r(t)*, for an increase in the short-rate itself under the following models:

(a) Vasicek model
(b) Cox, Ingersoll, and Ross model
(c) Hull and White model

[4]

[ii] Solve the stochastic differential equation under the one-factor Vasicek model to find the formula for the short-rate, *r(t).* [5]

[9]

Q.6) The probability density function *f(x)* of the 2 parameter Burr distribution is:

$$f(x;c,k) = ckx^{c-1}(1+x^c)^{-(k+1)}; where\ x > 0, c > 0\ and\ k > 0$$

[i] Prove that the cumulative distribution function of the above distribution is given by:

$$F(x;c,k) = 1 - (1+x^c)^{-k}$$

You have very limited knowledge of an investor's utility function.

[3]

[ii] Describe the following relationships between investment portfolios. [4]

(a) Absolute dominance
(b) First-order stochastic dominance
(c) Second-order stochastic dominance

Three investment portfolios A, B and C provide returns that follow Burr distribution with the following parameters.

Portfolio	***c***	***k***
A	1	1
B	1	2
C	2	1

[iii] Ascertain if an investor who prefers more to less would choose A over B? [4]

[iv] Show that the investor will not be able to choose between A and C based on first order stochastic dominance. [5]

[16]

Q.7) **[i]** Define Expected Shortfall below a certain level 'L' as a measure of risk? [1]

The return generated by an investment portfolio is modeled as follows:

$$R = 1\%X - 0.5\%(12 - X), where\ X \sim Binomial(12, 0.5)$$

The benchmark level is set at 0%.

[ii] Calculate the shortfall probability ***and*** expected shortfall below the benchmark level. [7]

The shortfall variance is defined as

$$\sum_{\mathrm{R_{min}}}^{L} (\mathrm{L} - \mathrm{r})^2 \mathrm{P}(\mathrm{R} = \mathrm{r})$$

[iii] Calculate the shortfall variance for the investment portfolio [4]

[iv] The Basel Committee on Banking Supervision is considering replacing Value-at-Risk as a measure of risk with Expected Shortfall. Comment on this move. [4]

[16]

Q.8) A market has 3 risky securities. The return on these securities is modeled using a statistical three-factor model of the following form:

$$R_i = a_i + b_{i,1}I_1 + b_{i,2}I_2 + b_{i,3}I_3$$

You are given the following information on the three securities and the three factors.

Security	**E[a_i]**	**Var[a_i]**	**$b_{i,1}$**	**$b_{i,2}$**	**$b_{i,3}$**
X	6%	9%%	1	0	0
Y	5%	4%%	0.5	0.5	0.5
Z	4%	1%%	0	1	0.5

Factor	**E[I_k]**	**σ_k**
I_1	**2%**	**1%**
I_2	**3%**	**2%**
I_3	**4%**	**4%**

[i] Calculate the mean, variance and covariance of the three securities. State any assumptions you make. [7]

Assume mean-variance portfolio holds.

Your aim is to calculate the proportions to invest in the three securities in a way that minimizes the risk. The expected return on the portfolio is set at E_p.

[ii] State the Lagrangian function to solve such minimization problems. Define all terms used. [3]

[iii] Derive the set of five simultaneous equations to solve this problem and represent these equations in matrix form.

You are given the following.

$$If\ A = \begin{bmatrix} 0.2\% & 0.01\% & 0\% & -8\% & -1 \\ 0.01\% & 0.185\% & 0.12\% & -9.5\% & -1 \\ 0\% & 0.12\% & 0.18\% & -9\% & -1 \\ 8\% & 9.5\% & 9\% & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

[7]

$$then\ A^{-1} = \begin{bmatrix} 86.21 & 172.41 & -258.62 & -55.17 & 5.22 \\ 172.41 & 344.83 & -517.24 & 89.66 & -7.55 \\ -258.62 & -517.24 & 775.86 & -34.48 & 3.33 \\ 55.17 & -89.66 & 34.48 & 14.69 & -1.28 \\ -5.22 & 7.55 & -3.33 & -1.28 & 0.11 \end{bmatrix}$$

[iv] Find the proportion of portfolio held in each asset in terms of E_p.

[4]

[21]

Q.9) **[i]** Define Semi-strong form efficiency.

There is substantial body of literature towards both, proving the existence of mispricings, in contravention of Efficient Market Hypothesis and proving the various forms of Efficient Market Hypothesis.

[1]

[ii] Suggest possible explanations to such categorical proof of mutually contradictory statements.

[3]

[4]

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