

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

26th November 2012

Subject CT1 – Financial Mathematics

Time allowed: Three hours (15.00 – 18.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1 Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception
- 2 Mark allocations are shown in brackets.
- 3 Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.
- 4 Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q.1) The force of interest $\delta(t)$, for time t is given by

$$\delta(t) = 0.08 + \frac{b}{t+1}$$

For an invested amount of ₹ 1000 at time $t = 0$, the accumulated value at time $t = 5$ is ₹ 1546.25.

Calculate the amount that should be invested at time $t = 2$ which will accumulate to ₹ 1000 at time $t = 8$.

[5]

Q.2) The fund value of an individual pension fund account was ₹95,000/- on 1st July 2006 and ₹1,65,065/- on 30th June 2012. During the period the investor made a payment of ₹9,000/- on 1st July 2009 and didn't withdraw any amount.

The fund manager found that between 1st July 2006 and 1st July 2012, the Money-Weighted rate of return for the account was 1.25 times its Time-Weighted rate of return.

[i] Calculate the fund value as on 30th June 2009. (5)

[ii] Which is the better measure of fund manager's performance - TWRR or MWRR? Why? (2) [7]

Q.3) An investment bank is issuing a bond with a coupon of 7.5% per annum payable quarterly in arrears and redeemable at par. The bond will be redeemed at the option of the borrower on any coupon payment date between 8 and 12 years from the date of issuance, both inclusive.

An investor only liable to income tax of 25% wants to invest in the bond.

Calculate the maximum price that the investor should pay for the bond if she wants to earn a net effective rate of return of 5.5% per annum.

[6]

Q.4) [i] If $i = 7.5\%$ per annum effective, calculate the following values up to four decimal places

[a] $\ddot{s}_{\overline{8}|}$ (2)

[b] ${}_2|\overline{a}_{\overline{5}|}$ (3)

[c] $a_{\overline{\infty}|}^{(4)}$ (2)

[ii] An investor wants to make following three investments

₹ 10,000 per annum payable continuously over a period of three years from now;

₹ 10,000 per annum payable monthly in advance, starting from beginning of 2nd year for a period of two years; and

₹ 10,000 per annum payable half yearly in advance, starting from beginning of 3rd year for a period of one year.

Calculate the accumulated amount at end of the third year from now at an interest rate of 10% per annum convertible half yearly. (8) [15]

Q.5) [i] Describe the investment characteristics of

[a] Eurobonds

[b] Short interest rate futures (6)

[ii] Currently the one year spot yield is 7% per annum effective. Also

$$f_{1,4} = 7.5\% \text{ per annum effective}$$

$$\text{and } f_{5,5} = 9\% \text{ per annum effective.}$$

Calculate the 5 year and 10 year spot yields up to four decimal places and hence comment on the shape of the yield curve in the light of expectation theory of interest rate. (4)

[iii] An ordinary share pays annual dividends. A dividend of ₹1 per share has just been paid. Dividends are expected to grow by 2% next year and by 4% the following year. Thereafter, dividends are expected to grow at 6% per annum compound in perpetuity. Currently the price of the share is ₹75 per share.

Calculate the forward price of the share at the end of 5th year just after payment of the dividend then due, assuming a risk free interest rate of 7% per annum payable continuously. (5) [15]

Q.6) The investment returns i of a mutual fund in different years are independent and identically distributed. $(1+i)$ follows a log normal distribution with mean 1.06 and standard deviation 0.02.

- [i] For an investment of ₹1 at time $t=0$, calculate the mean and standard deviation of the accumulated amount at time $t = 4$. (5)

Mr. ABC wants to invest ₹15,000 at time $t = 0$ and ₹12,000 at time $t = 1$ in the fund and wishes to withdraw the accumulated amount at time $t = 5$.

- [ii] Calculate the standard deviation of the accumulated amount at the end of 5th year. (6) [11]

Q.7) A film producer from Bollywood wants to build a studio. The cash flows associated with the project are as follows:

- [i] During the first three years, capital expenditure of ₹ 7.5 Crores, ₹ 5 Crores and ₹ 2.5 Crores are expected to be incurred at start of the respective years.

- [ii] From the fourth year onwards there is expected to be an increasing stream of income occurring at the mid of each year. The income during the fourth year will be ₹ 3 Crores which is expected to increase by ₹ 50 lakhs in each subsequent year.

- [iii] From the fifth year onwards the producer will have to make an increasing stream of payment for 5 years towards repayment of a loan. The amount of first payment will be ₹25 lakhs in the fifth year and it will increase at a rate of 5% per annum subsequently. These payments will be made at the beginning of each year starting from the fifth year.

- [iv] The studio will be sold at the end of 10th year at a price of ₹ 17 Crore.

Determine whether the producer should invest in the project if she is looking for at least 14% per annum effective return. [9]

Q.8) Exactly one year ago an actuarial student had taken an Educational Loan of ₹5,00,000/- from a bank, repayable over a period of 5 years in equal monthly installments in arrears. The interest rate charged by the bank is subject to change in future at the discretion of the bank. It was 10% per annum effective on the date of loan inception.

- [i] Calculate the level monthly installment payable by the student. (2)

Just after the payment of the 12th monthly installment the bank increases the interest applicable on the loan to 12% per annum effective.

- [ii] If the student doesn't want to change the monthly installment,

- [a] Calculate the remaining number of months over which the student will be required to repay the outstanding loan amount. (4)

- [b] Assuming all the installment payments are made at the month end calculate the amount of the reduced last payment. (2)

- [iii] After increasing the interest rate the bank offers an option for a payment holiday for the next one year. Under this option, the student does not have to pay any installment for the next one year and the interest on outstanding loan at the beginning of the payment holiday will accumulate during the period. If the student exercises this option but does not want to increase the term of the loan, calculate the revised level monthly installment amount that she will have to pay.

(3) [11]

Q.9) Six years ago a finance company offered a hybrid financial instrument for a 6 year term. The investor had to pay a single lump sum payment at the beginning of the term of which 25% would be invested in equities and the remainder would be invested in fixed interest securities at par. The fixed interest securities had a coupon of 7.5% per annum payable annually in arrears and were redeemable at 105% of invested amount at the end of 6 years.

An investor invested ₹6000 in this instrument. He received dividend on the equity portion as follows:

Time of dividend payment	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6
Dividend in ₹ per unit of share	0.35	0.4	0.475	0.5	0	0.45

The price for each unit of share was ₹12 at the time when the investor purchased the instrument and the price is ₹37 now. Assume that all shares are sold now at the prevailing price. The investor is not subject to any tax.

Calculate the number of share units the finance company purchased for the investor.

Calculate the IRR for the investor on this contract, to the nearest integer.

[9]

- Q.10) [i]** State Redington's Conditions for immunisation of a portfolio of assets and liabilities against small changes in interest rates. (2)

A pension fund has following liabilities

₹7 lakhs payable at the end of 6th year,
₹1 lakh payable at the end of 9th year,
₹2 lakhs payable at the end of 11th year.

The fund holds the following two assets to meet the liabilities

Investment A which provides income of ₹15,000 payable at the end of each year for the next five years and additionally a lump sum of ₹3 lakhs payable at the end of 5th year.

Investment B which is a zero coupon bond which pays a lump sum of ₹R at the end of n years (where n is not necessarily an integer).

The interest rate is 7% per annum effective.

- [ii] Calculate the net present value and volatility of the liability cash-flows. (4)
- [iii] Determine the values of ₹R and n such that the first two conditions of Redington's immunisation theory are satisfied. (6) **[12]**
