# **Institute of Actuaries of India**

# Subject ST8 – General Insurance : Pricing

# **May 2013 Examinations**

# **INDICATIVE SOLUTIONS**

# Solution 1 :-

- Liability insurance is a long tail business. Hence a claim might occur now, but may be reported years from now. So the actual loss ratios may take many years to develop completely. Therefore we may not have the opportunity to adjust the next year's premium on the basis of losses or profits made in the last year.
- Claims for liability insurance can be large. Therefore, mispricing it may lead to us suffering large losses which we cannot recover from even if we price correctly in future.
- One year's experience may not be a sufficient proof for future.
- If we under-price, we may make more losses than the company can bear, even if it is only one year.
- We might over price, and hence lose market share. Lower than expected sales might mean that our other assumptions, like expenses, may also not be validated.
- We may not be allowed by regulation to re-price, or if allowed to do so, we may not be able to re-price sufficiently to 'correct' our rates if we got them very wrong in the first place.
- Even if we are allowed to re-price, the market may not accept the rate revision.

#### [Total 3 Marks]

# Solution 2 :-

#### i & ii :-

	Quota S	ihare	Surp	Surplus			
Risk	Premium [QS% * Premium]	Claims [QS% * Claim]	Premium [(EML – Max retention / EML) * Premium]	Claims [(EML – Max retention / EML) * Claim]			
	100	3,750	100	3,750			
1	(=50%*200)	(=50%*7,500)	(=5,000/10,000*200)	(=5,000/10,000*7,500)			
	188		Cannot be placed since EML	> Max Ret * (Max lines +1)			
2	(=50%*376)	-	(27,900>5,000*5)				
	270	9,000	360	12,000			
3	(=50%*540)	(=50%*18,000)	(=10,000/15,000*540)	(=10,000/15,000*18,000)			

Assumption: the insurer uses maximum retention for each risk.

#### Alternatively

	Quota S	hare	Surplus			
Risk	Premium [QS% * Premium]	Claims [QS% * Claim]	Premium [Max lines/ (Max lines +1) * Premium]	[Max lines/ (Max lines +1) * Claim]		
	100	3,750	160	6,000		
1	(=50%*200)	(=50%*7,500)	(=4/5*200)	(=4/5*7,500)		
	188		Cannot be placed since EML	. > Max Ret * (Max lines +1)		
2	(=50%*376)	-	(27,900>	5,000*5)		
	270	9,000	432	14,400		
3	(=50%*540)	(=50%*18,000)	(=4/5*540)	(=4/5*18,000)		

Assumption: the insurer uses maximum lines for each risk.

[Total 3 Marks]

#### Solution 3 :-

Various checks you will perform on the office premium rates for reasonableness, consistency and market acceptability:

- Progression of premium rates across different levels of same rating factor
- Comparison with previous rates and reasonableness of rate change for each risk cell
- Comparison with competition
- Comparison with other coverage. Higher coverage should have higher premium
- Any major changes in NCD levels
- Differences in premium rates by different channels
- Regulatory compliance
- Level of cross subsidy. Claim ratio for each risk cell, at various groups and at portfolio level
- If there are changes in the rating factors or levels then before and after scenarios in like to like risk cells

# Solution 4 :-

#### i. Differences:

- Classical credibility theory does not require information regarding the distribution of the underlying parameter,  $\theta$ .
- Bayesian credibility never reaches the value 1, the asymptote.
- Where estimates of k are available, Bayesian credibility generates the more accurate results.
- Classical credibility is simpler to calculate and easier to explain to non-actuaries.

(2)

**ii.** Credibility is based on the hypothesis that the underlying model is correct, and then uses information from other sources to determine 'how correct' it is. Therefore, the statistic for measurement, i.e. the credibility factor, is calculated using the model. In other words, you assign full credibility to your model, until proven otherwise.

(2)

#### [Total 4 Marks]

#### Solution 5 :-

i. Let X be the expected loss ratio for segment 2. Then,

Office premium for segment 1 = 5000 / 55% = 9090.9. Office premium for segment 2 is 8000/X.

Expected loss for the portfolio = 5000\*75% + 8000\*25% = 5750.

Expected premium for the portfolio = 75% \*9090.9 +25% \* 8000/X = 6818.2 + 2000/X

65% = 5750 / (6818.2 + 2000/X)

 $\Rightarrow 6818.2 + 2000/X = 5750/65\% = 8846.2$ 

 $\Rightarrow$  1/X = (8846.2 - 6818.2) / 2000 = 101.4%

$$\Rightarrow$$
 X = 1/101.4% = 98.8%

(2)

- **ii.** The situations under which one segment may be made to cross subsidize another segment in a portfolio:
  - Premium paying capacity of the segment is higher
  - Management discretion to make a segment loss-leader to capture niche market
  - Regulatory constraint on premium rates
  - Prevailing market rates
  - Low volatility in claim experience in a segment may require a lower contingency margin compared to another
  - Premium rate kept same across risk groups for administrative ease even though the loss ratios are different for risk groups

(3)

# [Total 5 Marks]

#### Solution 6 :-

#### i. Claimant

- time value of money for the claimant may be higher.
- he might be expecting that the actual interest rate will be higher than the one used to discount the cashflows.
- Less hassle. For example, the court may require a proof of disability / unemployibility to be submitted before each payment.
- The lump sum may be useful for paying the expenses for treatment.

#### Company

- would prefer to settle any liabilities immediately so that they do not need to maintain reserves.
- would save on expenses of having to make structured remittances if paying in lump sum, particularly given the long term liability, future expenses may bite due to expense inflation.
- the company may be expecting that the actual interest rate will be lower than the one used to discount the cashflows.

(3)

#### ii. Possible impact:

- Not many claims end up in court. Thus this may be a one-off result.
- However, usually, decisions in court impact how future claims are settled even outside the court.
- In such a case, though, the claimant and the insurer may both prefer a lump sum payment. Hence, if settled outside the court, the court's decision may not impact the outside court settlement.
- If we did want to factor the small chance that the claim goes to court we would need to make an assumption regarding what probability can be assigned to the

cases reaching the court. And for this proportion, we might wish to assume a cashflow structured payment instead of a lump sum payment. Long-term expense and inflation assumptions related to this would be required. Reserves would need to be held till the last payment is made.

(2)

[Total 5 Marks]

#### Solution 7 :-

**i. Individual risk model**: Under the individual risk model the total claim amount S payable during a specified period in respect of a block of policies is:

$$\mathbf{S} = \mathbf{X}_1 + \mathbf{X}_2 + \ldots + \mathbf{X}_n$$

where X<sub>i</sub> is the claim amount payable during the period in respect of risk i and

n is the number of risks.

Assumptions:

- a) Risks are independent
- b) Claim amount for risks need not be IID
- c) Number of risks does not change over time
- **d**) Number of claims from each risk is either 0 or 1, with probability of claim p<sub>i</sub>, which may vary for each risk

**Collective risk model**: Under the collective risk model the total claim amount S payable during a specified period in respect of a block of policies is:

 $S = X_1 + X_2 + \ldots + X_N$ 

where  $X_i$  is the claim amount payable during the period in respect of the i th claim and N is the (random) number of claims during the period.

#### Assumptions:

- a) Claim Amount X<sub>i</sub> are IID
- **b**) The  $X_i$  s and N are independent

(4)

**ii.** Scenario for applying Individual risk model: Modeling total claim amount in a group term life account OR modelling total claim amount from a motor fleet where multiple claims under a vehicle are summed-up to get a total claim per vehicle.

Scenario for applying collective risk model: Modelling total claim amount from a motor fleet.

(2)

[Total 6 Marks]

# Solution 8 :-

i.

# a) Advantages:

- Does not take on the risk.
- Can appear to offer a comprehensive portfolio without taking on the risk.
- Gets a fee without writing the underlying risk.
- May be able to develop an expertise in a new product.
- May be able to retain small risks and hence benefit from the underwriting profits.
- May benefit from tax advantages.

#### **Disadvantages:**

- Usually does not obtain any substantial proportions of the underwriting profits from writing the business other than the commission.
- It remains liable to the policyholder.
- It takes on additional counter-party risks.
- It may be taking on regulatory risks, in case regulators view regarding fronting becomes adverse.

# b) Advantages:

- Will be able to write risk that it otherwise may not be able to due to licensing constraints.
- Tax advantages.
- Allowing the reinsurer to take advantage of the insurer's (potential) higher rating.
- Can write business without needing to develop a distribution network.

#### **Disadvantages:**

- Takes on counter party risk.
- It may have lesser control on the distribution channels and hence the quality of business.

(2)

(3)

#### ii. Regulator:

- May not be able to apply controls, checks and balances on the company that is actually assuming the risk.
- It may appear to be 'unfair' to the policyholder since they would indirectly be funding the fees to the fronting insurer.
- It may also appear to be 'unfair' to the policyholder since they are not aware who is actually assuming their risk.

(2) [Total 7 Marks]

#### Solution 9 :-

#### **Quota Share**

- Since we are a small company. Hence, the large claims must be creating substantial strain on capital and solvency margins. Either Surplus or XS of loss will allow us to cede higher claims above the retention limits we are comfortable with, and hence allow us to get higher stability in results.
- + We have only started selling the product last year, so we not have sold enough business as yet for the portfolio to stabilise. It might do so as we write more business, and so our experience may no longer be erratic.
- + Also, by having a quota share agreement, the reinsurer shares our fortune on all cases, small and large, and hence might be giving us considerable guidance on underwriting and claims procedures.
- Using Surplus or Excess of Loss reinsurance might allow us to write much larger risks with a potentially smaller retention, allowing us to expand our business and volumes and hence bring even more stability to our portfolio.

#### <u>Surplus</u>

- + With Surplus reinsurance we will be able to choose a retention limit (subject to limits) for each risk individually, allowing us to fine tune our experience and profitability. For example, since our experience on small claims does not seem to be bad we could choose to retain smaller risks entirely, subject to the maximum retention negotiated under the Surplus agreement.
- However, Surplus reinsurance will require more administrative resources, since each risk will be reinsured with different retention. The staff will need to be trained, and IT models will need to be developed to cope with this.
- Also, if the Estimated Maximum Loss for a particular risk is higher than the maximum retention and maximum capacity that the reinsurer is willing to offer, then we might be forced to retain that risk entirely, unless another facultative reinsurer arrangement exists.

#### **Excess of Loss**

- + Excess of Loss insurance might be more useful in this scenario, since we could buy various layers of excess of loss reinsurance, with the top layer offering unlimited cover particularly since we are facing a few large claims.
- However, both quota share and surplus, would offer override commission and return commissions which would help with the initial cashflow and new business strain, which is usually not offered under Excess of Loss.
- Quota Share and Surplus being proportional reinsurance, both may be written on a 'policy incepting' basis, while a non-proportional reinsurance like 'Excess of Loss' may be written on a 'loss occurring' or 'claims reported' basis. This change in basis may cause problems in administration.

#### **Other considerations**

- The local regulator may actively encourage or discourage one type of reinsurance over the other. We would need to consider this before deciding on which reinsurance structure to use.
- Both Surplus and XS of Loss are very similar arrangements and may have very similar results for our business. A cost benefit analysis would be required to determine which would be more benefitial financially.
- Reinsurance commission may have been negotiated for a longer duration under the current treaty. The reinsurer might now offer lower commission after analysing our experience under surplus arrangement, or may offer reinsurance rates that are less beneficial than the previously negotiated commission rates.
- + We may wish to consider having a combination of Surplus and Excess of Loss reinsurance structures allowing us more protection and higher flexibility.
- The reinsurer may be more involved with the underwriting and claims procedures for larger risks under Excess of Loss and Surplus reinsurance since they would be exposed for a higher proportion of the risk. For Quota share, they may be proving guidelines to follow without actively getting involved in the procedures.

#### [Total 7 Marks]

#### **Solution 10 :-**

- Multivariate analysis should be performed to check the exact impact of State G on pure claim cost. The overall 10% higher risk premium may be due to other factors such as
  - Type of Cover
  - Sum insured
  - Vehicle types
  - o Vehicle age-mix
  - Age / gender of drivers
  - Types of use of vehicle
  - Policy excess or any other options taken.
- To come up with a change in premium rate, overall combined ratio should be looked at and not just pure claim cost. Other factors such as expense ratio (including mix of new and renewal business) and commission ratio should also be looked at which impact the overall profitability of business from State G.
- Even though pure claim cost is higher in state G, the commission rate may be lower due to a different channel mix. In that case, increasing premium by 10% will not be correct.
- The volume of business and claims from State G should be carefully considered. As the company started selling motor insurance only 5 years back, the exposure base in State G may not be credible. The claim experience may also get impacted due to some large claims.
- If the experience is not credible, it may be better to wait till a credible experience base is built. Credibility weighted estimates should be used till such time.

- There may be more instances of fraud in state G compared to other states. in such a case, increasing premium rates may not a right solution and fraud control measures should be applied to contain higher claim cost from this state.
- The premium rate of other insurers (possibly larger) should be looked at to check if they have also a higher premium rate in that state. If possible, claim ratio of these insurers in the state should also be looked at.

# [Total 8 Marks]

#### **Solution 11 :-**

#### i. Risk Factors

- A risk factor is a factor that is expected to influence the intensity of risk.
- For practical reasons it may not be possible to use all risk factors as rating factors. Rating factors need to be measurable, verifiable and objective.
- It may not be possible to measure some of the risk factors. For example, in commercial property insurance, "how well the company is run" may be a risk factor, but cannot be a rating factor. It may, however, be an underwriting factor.
- It may not be possible to measure the impact on risk of all the risk factors, and so some of the less critical risk factors may be ignored.
- In some legislation it may not be legal to use some of the risk factors as rating factors. E.g. Gender can no longer be used in European markets.
- It may not be practical to collect all the information required to use all the risk factors as rating factors.
- Some risk factors may not be used as rating factors for competitive reasons.

#### ii.

# a) Pet insurance:

- Animal breed
- Pet's age
- Pet's pedigree
- Pet's past medical history
- Any Excess applying
- Any deductibles applying
- Residential address of the pet owner
- Duration of cover
- Vaccination history
- Pet's sex
- Value of the pet
- Indoor / outdoor pet
- Income bracket of the pet owner

(4)

# b) Extended Warranty Cover:

- Make of machine
- Model of machine

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- Length of manufacturer's guarantee
- Term of warranty
- Price
- engine / motor size and /or type
- Deductible

(2) [Total 8 Marks]

#### **Solution 12 :-**

#### i. Possible reasons:

- The company is younger, so it might have more up to date information systems, using the latest technology.
- Its spending or focus on data systems may have been different from the bigger company.
- It is a smaller company and newer, so data may be lesser, so a less robust information system may suffice.
- Newer company may not have anticipated the granularity of info required since it has less experience.
- It may be covering a different segment, requiring a different (lesser or greater) detail in the information system.
- The involvement of various departments (e.g. the actuarial department) may have differed between the two companies at the time when the system was being developed. Hence, the system might be catering more to the needs of one department than the other in the two companies.
- There may have been legacy issues in the data systems used by either of the companies. This is more likely for the older company.

(3)

#### ii. Factors:

- The cost of the systems;
- Comprehensiveness of data capture;
- Robustness of the systems;
- Which system has more data associated with it, which will need to be migrated.
- Which system meets the need of the company going forward more closely given its business plans.
- Appropriateness from the business environment point of view, type and nature of business, geography, regulations etc.
- Which system is more user-friendly.
- Which system is more efficient.
- Track record or error records from each of the systems.

(3)

# iii. Steps:

- A decision would need to be taken with regards to whether the existing business needs to be migrated.
- If the older company was mostly writing one year-renewable contracts which are not long tail businesses, it may choose to run off the existing business on the old system and only migrate the new business to the new system. If this happens, then
  - The company will have to maintain both the systems until all the claims possible have been made with reasonable certainty.
  - The company will have to make 'adjustments' / approximations to incorporate both the systems in combined reports.
- If the existing business is to be migrated also, for a short while both the systems will need to be used simultaneously until:
  - users become familiar with the new system
  - The new system has been tested and optimised for the use of the new company
  - $\circ~$  The results of the new system have been verified.
- Checks on the data will need to be put in place for the migration.
- We would need to map the fields for the two information systems to figure out which fields need to be added and / or subtracted. We could use this opportunity to take a forward looking approach and add more fields that might be of use in future.
- If fields are to be added, then we would need to figure out whether the data required to populate those fields is already being collected in the application forms or not. If not, then the application forms will need to be modified to collect required information. This might cause expenses and logistic concerns.
- Staff would need to be trained to use the new systems.
- Any systems that integrate with the information system would require a change. For example, an expense management tool, or a pricing tool, or a business monitoring tool that was directly linked to the current information system would need to be modified.

(5) [Total 11 Marks]

# Solution 13 :-

i.

Year	Earned Premium (A)	Attritional claims (B)	Large claims ©	Natural Catastrophic claims (D)	Rate changes (E)	Inflation (F)
2008	50	27.5	0	0		
2009	60	35	16.0	0	-2%	5%
2010	70	49	10	21.5	10%	5%
2011	80	52.0	0	0	5%	5%
2012	90	50.0	4	0	0%	5%
2013					8%	5%

Year	Rate change index	Inflation index	Adjusted Premium	Adjusted Attritional Claims	Adjusted Large Claims	Attritional Claim ratio	Adjusted CAT claims	Total claim ratio	Reinsu'er's profit without profit commission
2008	100.00	100.00	78.0	35.1	-	45.0%	-	45.0%	30.0%
2009	98.00	105.00	91.0	42.5	19.4	46.8%	-	68.1%	6.9%
2010	107.80	110.25	91.9	56.7	11.6	61.7%	24.9	101.4%	-26.4%
2011	113.19	115.76	95.3	57.3	-	60.2%	-	60.2%	14.8%
2012	113.19	121.55	102.1	52.5	4.2	51.4%	-	55.6%	19.4%
2013	122.25	127.63	100.0	-	-		-	0.0%	

Rate change index for Year X = Rate change index for year (X-1)\*(1+Rate change% for year X). 100 for 2008.

Inflation index for year X =Inflation index for year  $(X-1)^*(1+inflation for year X)$ . 100 for year 2008.

Adjusted premium for year X = Earned Premium for year X \* (2013 rate change index / rate change index for year X) \* (2013 inflation index / inflation index for year X)

Adjusted claims (both Attritional and Large) for year X = Unadjusted claims for year X \* (2013 inflation index / inflation index for year X)

Average attritional claim ratio = sum of adjusted attritional claims (from 2008 to 2012) / sum of adjusted premium (2008 to 2012) = 53.3%

Average large claim ratio = sum of adjusted large claims (from 2008 to 2012) / sum of adjusted premium (2008 to 2012) = 7.7%

CAT claim ratio = 3%

Expense ratio = 5%

Minimum Profit margin = 4%

Maximum ceding commission payable = 1-53.3% - 7.7% - 3% - 5% - 4% = 27%

(7)

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1	1	
-	-	•

Year	Adjusted CAT claims	Total claim ratio	Reinsurer's profit without profit commission
2008		45.0%	30.0%
2009		68.1%	6.9%
2010	24.9	101.4%	-26.4%
2011		60.2%	14.8%
2012		55.6%	19.4%

Total claim ratio above is = sum of adjusted attritional, large and CAT claims over adjusted premium

Profit = 1 - total claim ratio - expense ratio - ceding commission

(2)

#### iii.

Year	Total claim ratio	Expense ratio	Ceding commission	Profit before profit commission (PC)	PC @ 25%	Profit after PC
2008	45.0%	5.0%	5.0%	45.0%	11.3%	33.8%
2009	68.1%	5.0%	5.0%	21.9%	5.5%	16.4%
2010	101.4%	5.0%	5.0%	-11.4%	0.0%	-11.4%
2011	60.2%	5.0%	5.0%	29.8%	7.5%	22.4%
2012	55.6%	5.0%	5.0%	34.4%	8.6%	25.8%

Profit before PC = 1 - total claim ratio - expense ratio - ceding commission

PC = 25% \* profit, if profit is more than 0%

Profit after PC = Profit before PC – Profit commission

(3)

iv. Standard deviation of profits in (ii) is 21.5% while it is 17.3% in (iii). Profit commission helps in reducing the volatility of profits for reinsurers. (2)

#### [Total 14 Marks]

#### **Solution 14** :-

- i. Data fields you will require to perform the analysis:
  - Risk identifier
  - Sum insured /PML of the risk
  - Type of risk
  - Geographic territory of risk
  - Claim identifier
  - Cause of loss
  - Gross claim amounts Paid
  - Gross claim reserve

(4)

- Claim event date
- Dates of payments
- Claim status (open / closed)

ii.

- All companies may not contribute
- Coding systems may be different across companies (for example for type of risk and territory)
- Data may be outdated due to delays in data collection
- Claim practices may be different (for example for payment and reserving)
- Data may not be as detailed as required
- Data quality will depend on the quality of data systems used
- Policies / coverage sold by companies may be different

(3)

iii.

We need an estimate for G(10%) = LEV<sub>Y</sub>(10%)/E(Y), where Y = Claim Amount / PML

Let there be N claim amounts in the data. Let  $X_i$  and  $M_i$  be the i-th claim amount and PML amount respectively and  $Yi = X_i / M_i$ .

Then G(10%) = 
$$\left(\frac{\sum_{OverN} Maxmium(Y_i, 10\%)}{\sum_{OverN} Y_i}\right)$$

Alternatively,

$$G(10\%) = \left(\frac{\sum_{OverN} Maxmium(X_i, 10\% * M_i)}{\sum_{OverN} X_i}\right)$$

(	4)
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iv.



v.

• Exposure curve should be monotonously increasing with decreasing rate of increase (second order derivative should be negative) (1)

vi.

• 
$$G(x) = \left(\frac{1-0.1^x}{1-0.1}\right) \text{ for } 0 \le x \le 1.$$

Deductible as % of PML = 25%

Limit as % of PML = 50%

G(50%) = 0.7597 and G(25%) = 0.4863

So, risk premium as a percent of original risk premium = 0.7597 - 0.4863 = 0.2734= 27.34%

(2)

[Total 15 Marks]

#### \*\*\*\*\*