Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

May 2013 Examinations

INDICATIVE SOLUTIONS

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1 :

(i) $A_x = E(X)$ where

$X = v^{K_x+1}$ if $K_x < n$

v^{K_x+1} if $K_x \geq n$

$A^1_{x:\overline{n|}} = E(Y)$ where

$Y = v^{K_x+1}$ if $K_x < n$

0 if $K_x \geq n$

$n|\,A_x = E(Z)$ where

$Z = 0$ if $K_x < n$

v^{K_x+1} if $K_x \geq n$

$Y + Z = v^{K_x+1} + 0 = v^{K_x+1}$ if $K_x < n$

$0 + v^{K_x+1} = v^{K_x+1}$ if $K_x \geq n$

⇨ $Y + Z = X$
⇨ $E(X) = E(Y) + E(Z)$
⇨ $A_x = A^1_{x:\overline{n|}} + n|\,A_x$

Interpretation: A whole life benefit is equal to a term assurance for n years (which pays out on death in the first n years) plus a benefit covering the whole of the remainder of the policyholder's life, provided the policyholder survives for n years.

(ii) $a_x = E(a_{\overline{K_x|}}) = \sum_{k=0}^{\infty} a_{\overline{k|}}\ {}_{k|}q_x$

$$= 0 \times {}_{0|}q_x + v \times {}_{1|}q_x + (v + v^2) \times {}_{2|}q_x + (v + v^2 + v^3) \times {}_{3|}q_x + \ldots$$

Reversing the order of summation:

$a_x = v\,({}_{1|}q_x + {}_{2|}q_x + \ldots) + v^2\,({}_{2|}q_x + {}_{3|}q_x + \ldots) + \ldots$

$$= v \sum_{k=1}^{\infty} {}_{k|}q_x + v^2 \sum_{k=2}^{\infty} {}_{k|}q_x + \ldots$$

$$= \sum_{j=1}^{\infty} \left(\sum_{k=j}^{\infty} {}_{k|}q_x \right) v^j$$

$$= \sum_{j=0}^{\infty} \left(\sum_{k=j+1}^{\infty} {}_{k|}q_x \right) v^{j+1}$$

But, for example, $\sum_{k=1}^{\infty} {}_{k|}q_x$ is the probability that the life dies after 1 year (i.e. is still alive after 1 year), and which we can therefore write as ${}_1p_x$.

$$\therefore \sum_{k=j+1}^{\infty} {}_{k|}q_x = {}_{j+1}p_x$$

$$\Rightarrow \quad a_x = \sum_{j=0}^{\infty} {}_{j+1}p_x v^{j+1} = \sum_{j=1}^{\infty} {}_j p_x v^j$$

Interpretation: The present value of each annuity payment is conditional on whether the policyholder is alive or not at the time the payment is due. If the policyholder is alive, then the present value is v^j, otherwise 0. The expected present value of the payment is then v^j multiplied by the probability of being alive at this point, ${}_jp_x$. Summing over all future years gives the expected present value of all the future benefit payments.

[Total-6]

Solution 2 :

(i) Present Value = $\begin{cases} 10{,}00{,}000 \times V\text{^}Ty & , Ty>Tx \\ 0 & , Ty<=Tx \end{cases}$

(ii) Expected Present Value = 10,00,000$\int_{t=0}^{\infty} v\text{^}t$ × tqx × tpy × μy+tdt,

Where tqx is probability that life aged x will die in time t

$_tp_y$ is probability that life aged y will be alive till time t

v^t is the discounting factor

μ_{y+t} is the force of mortality for life aged y+t

[Total-4]

Solution 3 :

(i) Schemes usually allow members to retire on grounds of ill-health and receive a pension benefit after a minimum length of scheme service.

To prevent selection against other scheme members, entitlement usually depends on evidence of ill-health being provided.

Benefits are usually related to salary at the date of ill-health retirement in similar ways to age retirement benefits.

However, pensionable service is usually more generous than under age retirement with years beyond those served in the scheme being credited to the member e.g. actual pensionable service subject to a minimum of 20 years, or pensionable service that would have been completed by normal retirement age. A lump sum may be payable on retirement and a spouse pension on death after retirement.

(ii) $\int_{t=0}^{\omega} 1 \times \frac{(al)_{x+t}}{(al)_x} \times (a\mu)^i_{x+t} \times v^t \times \bar{a}^i_{x+t}\, dt$

Where

1. $\frac{(al)_{x+t}}{(al)_x}$ is the probability that an active member aged x at the valuation date will alive till age x+t

2. $(a\mu)^i_{x+t}$ is the probability that an active member aged x at the valuation date will retire at age x+t

3. v^t is the discounting factor
4. $\bar{a}^i_{x+t}$ is the annuity payable continuously after ill health retirement

[Total-5]

Solution 4 :

Reserve at time t + Interest + Premiums received during (t,t+h)

= Reserve for survivors at time (t+h) + Benefits paid during (t,t+h)

⇨ $${}_t\bar{V}^1_{x:\overline{n}|} + \delta h\,{}_t\bar{V}^1_{x:\overline{n}|} + h\bar{P}^1_{x:\overline{n}|} = (1-h\mu_{x+t})\,{}_{t+h}\bar{V}^1_{x:\overline{n}|} + h\mu_{x+t} + o(h)$$

o(h) term is needed as the interest and survival probability are only accurate to first order.

Rearranging the above equation:

$$\frac{{}_{t+h}\bar{V}^1_{x:\overline{n}|} - {}_t\bar{V}^1_{x:\overline{n}|}}{h} = \delta\,{}_t\bar{V}^1_{x:\overline{n}|} + \bar{P}^1_{x:\overline{n}|} + \mu_{x+t} \times {}_{t+h}\bar{V}^1_{x:\overline{n}|} - \mu_{x+t} + \frac{o(h)}{h}$$

Taking the limit as h tends to zero:

$$\lim_{h\to 0}\frac{{}_{t+h}\bar{V}^1_{x:\overline{n}|} - {}_t\bar{V}^1_{x:\overline{n}|}}{h} = \delta\,{}_t\bar{V}^1_{x:\overline{n}|} + \bar{P}^1_{x:\overline{n}|} + \mu_{x+t} \times {}_t\bar{V}^1_{x:\overline{n}|} - \mu_{x+t}$$

i.e. $$\frac{\partial}{\partial t}\,{}_t\bar{V}^1_{x:\overline{n}|} = (\mu_{x+t} + \delta)\,{}_t\bar{V}^1_{x:\overline{n}|} + \bar{P}^1_{x:\overline{n}|} - \mu_{x+t}$$

i.e. $$\frac{\partial}{\partial t}\,{}_t\bar{V}^1_{x:\overline{n}|} = \delta\,{}_t\bar{V}^1_{x:\overline{n}|} + \bar{P}^1_{x:\overline{n}|} - (1 - {}_t\bar{V}^1_{x:\overline{n}|})\mu_{x+t}$$

[Total-5]

Solution 5 :

Expected present value of the benefit Amount is given by:

= 100,000 $\int_0^{10} v^t \times {}_tp^{HH}_{50} \times (\mu_{50+t}+\sigma_{50+t})\, dt$

= 100,000 $\int_0^{10} v^t \times {}_tp^{HH}_{50} \times (0.003+0.005)\, dt$

= 800 $\int_0^{10} v^t \times {}_tp^{HH}_{50}\, dt$

We have ${}_tp^{HH}_{50}$ = Probability that a healthy life aged 50 is healthy at the age 50+t

= exp (-0.008t)

Then the EPV of benefit = 800 $\int_0^{10} \exp(-\delta t)$ × exp (-0.008t) dt

Here δ= In(1.08) = 0.076961

Then the EPV of benefit = 800 $\int_{t=0}^{10} \exp(-0.076961t)$ exp (-0.008t) dt

= 800 $\int_{t=0}^{10} \exp(-0.084961t)$dt

=5390

[Total-5]

Solution 6 :

At t=0, the present value of all future payments is:

$$S = \sum_{j=1}^{10000} Z_j$$

Where$Z_j$is the present value at t=0 for the payment to be made at the death of life numbered j.

Now, for each j, $E(Z_j) = \bar{A}_x = \int_0^\infty e^{-\delta t} e^{-\mu t} \mu dt = \frac{\mu}{\mu+\delta} = 0.4$

$$E(Z_j^2) = {}^2\bar{A}_x = \int_0^\infty e^{-2\delta t} e^{-\mu t} \mu dt = \frac{\mu}{\mu+2\delta} = 0.25$$

$\mathrm{Var}(Z_j) = 0.09$

$\therefore E(S) = 10,000 \times 0.4 = 4,000$ andVar(S) = 10,000 × 0.09 = 900

Let α be such that P(S ≤ α) = 0.95

i.e. $P\left(\frac{S-E(S)}{\sqrt{Var(S)}} \le \frac{\alpha - 4000}{30}\right) = 0.95$

By using normal approximation:

$$\frac{\alpha - 4000}{30} = 1.645$$

⇨ α=4,049.35 **[Total-6]**

Solution 7 :

ASSUMPTIONS TO BE MADE

• Decrements in each single-decrement table are uniformly distributed over each year of age.

• Decrements in the multiple-decrement table are uniformly distributed over each year of age.

q(21)w = (ad)(21)w ÷ (1-0.5 * (ad)(21)s) = 0.2196

q(21)s = (ad)(21)s ÷ (1-0.5 * (ad)(21)w) = 0.0772

q(22)w = (ad)(22)w ÷ (1-0.5 * (ad)(22)s) = 0.0234

q(22)s = (ad)(22)s ÷ (1-0.5 * (ad)(22)w) = 0.1013

New independent rates after improvement in withdrawals are calculated as:

q(21)w = 60% * 0.2196 = 0.1317

q(21)s = 0.0772 (same as old rates)

q(22)w = 60% * 0.0234 = 0.0141

q(22)s = 0.1013 (same as old rates)

New dependent rates after improvement in withdrawals are calculated as:

aq(21)w = q(21)w × (1-0.5 * q(21)s) = 0.1267

aq(21)s = q(21)s × (1-0.5 * q(21)w) = 0.0721

aq(22)w = q(22)w × (1-0.5 * q(22)s) = 0.0133

aq(22)s = q(22)s × (1-0.5 * q(22)w) = 0.1006

The revised table is given as:

Age	(al)x	(ad)s	(ad)w
21	1,000	72	127
22	801	81	11
23	710		

[Total-7]

Solution 8 :

(i) **Crude Mortality Rates:**

Advantages : Do not require population and deaths split by age

: Easy to calculate

Disadvantages: Not a reliable indicator as difference in the age structure between population gets lost

Standardised Mortality Rates:

Advantages : Takes into account population structure

: Reliable indicator of mortality rates

Disadvantages: requires age specific mortality rates for the population

: Time consuming and complicated method

(ii) **Standardised Mortality Ratio:**

Total Actual no. of death in Madagascar City = 130+322+564 = 1016

Now we need to calculate expected no. of deaths using standard population data.

For age 50, expected deaths = 15000 × 56,675 ÷ 35,00,000 = 242.89

For age 55, expected deaths = 12000 × 93,006 ÷ 32,00,000 = 348.77

For age 50, expected deaths = 9500 × 1,16,090 ÷ 22,00,000 = 501.30

Total expected no. of death in Madagascar City = 1092.96

SMR = 1016/1092.96 = 92.96%

[Total-7]

Solution 9 :

(i) For a term assurance contract, the expected cost of paying benefits increases over the term of the contract as the probability of death increases with age whereas the premiums are level.

This means that the premiums received in the early years of a contract exceed the benefits that fall due in those early years.

But in the later years, the premiums are too small to pay for the benefits.

It is therefore prudent for the premiums that are not required in the early years of a contract to be set aside, or reserved, to fund the shortfall in the later years of the contract.

(ii) The net premium reserves at times t and t+1 are related by the formula:

$({}_tV_x + P)(1+i) = q_{x+t}\, S_{t+1} + p_{x+t\,t+1}V_x$

where S_t is the death benefit payable at time t.

Rearranging this equation, we get:

$P = (S_{t+1} - {}_{t+1}V_x)vq_{x+t} + (v\,{}_{t+1}V_x - {}_tV_x)$

Here $S_{t+1} = {}_{t+1}V_x$ for t=0,1,2,…,n-1 and hence

$P = v\,{}_{t+1}V_x - {}_tV_x$

⇨ $v^t P = v^{t+1}{}_{t+1}V_x - v^t{}_tV_x$

Summing over t = 0,1,2,…,n-1 we get

$$P\sum_{t=0}^{n-1} v^t = v^n\,{}_nV_x - v^0\,{}_0V_x$$

⇨ $$P\ddot{a}_{\overline{n}|} = v^n\ddot{a}_{x+n} - v^0 \times 0$$

⇨ $$P = \frac{v^n\ddot{a}_{x+n}}{\ddot{a}_{\overline{n}|}} = \frac{\ddot{a}_{x+n}}{\ddot{s}_{\overline{n}|}}$$

[Total-8]

Solution 10 :

(i) It is a principle of prudent financial management that once sold and funded at outset a product should be self-supporting.

This implies that the profit signature has a single negative value (funds are provided by the insurance company) at policy duration zero. This is often termed "a single financing phase at the outset".

Many products produce **"naturally"** profit signatures which usually have a single financing phase. However, some products, particularly those with substantial expected outgo at later policy durations, can give profit signatures which have more than one financing phase. In such cases these later negative cashflows should be reduced to zero by establishing reserves in the non-unit fund at earlier durations. These reserves are funded by reducing earlier positive cashflows.

(ii) The cashflows in 5 years are given as : (-10, -20, -5, 15, 40)

Since cashflows in year1, year2 and year 3 are negative. Hence, we need to set non unit reserves at the end of year 2 and at the end of year 1.

${}_2V$ = 5/(1.05) = 4.76

${}_1V$ = (${}_2V \times {}_1p_{51}$+ 20)/(1.05) = 23.57

(iii) Before zeroisation, NPV is given by:

$$\text{NPV} = \frac{C1 \times 0p50}{1.1^1} + \frac{C2 \times 1p50}{1.1^2} + \frac{C3 \times 2p50}{1.1^3} + \frac{C4 \times 3p50}{1.1^4} + \frac{C5 \times 4p50}{1.1^5}$$

$$= \frac{-10 \times 1.0000}{1.1^1} + \frac{-20 \times 0.99749}{1.1^2} + \frac{-5 \times 0.99469}{1.1^3} + \frac{15 \times 0.99155}{1.1^4} + \frac{40 \times 0.98805}{1.1^5}$$

= -9.091 -16.487 -3.373 + 10.159 + 24.540
= 5.383

After zeroisation, cashflow at year 1 is
= C1 - ${}_1V$* 1p50
= -10 – 23.57*0.99749

= -33.51

After zeroisation, NPV is given by:

NPV $= \frac{C1 \times 0p50}{1.1^1} + 0 + 0 + \frac{C4 \times 3p50}{1.1^4} + \frac{C5 \times 4p50}{1.1^5}$

$= \frac{-33.51 \times 1.0000}{1.1^1} + 0 + 0 + \frac{15 \times 0.99155}{1.1^4} + \frac{40 \times 0.98805}{1.1^5}$

= -30.464 + 10.159 + 24.540

= 4.234

(iv) NPV before zeroisation is expected to be higher than NPV after zeroisation.

The reason is that zeroisation of reserves results in delay of profit emergence.

Also, RDR is greater than accumulation rate.

[Total-10]

Solution 11 :

(i) The death strain at risk for a policy for year $t + 1$ (t = 0, 1, 2) is the excess of the sum assured (i.e. the benefits payable on death during the year $t + 1$) over the end of year provision.

i.e. DSAR for year $t + 1 = S - {}_{t+1}V$

The expected death strain for year $t + 1$ (t = 0, 1, 2) is the amount that the life insurance company expects to pay in addition to the end of year provision for the policy.

i.e. EDS for year $t + 1 = q_{x+t}(S - {}_{t+1}V)$

(ii) At t=0, each life contributes 1,000,000 A_{45}. As there are 1,000 lives the fund value at t=0 is:

$F_0 = 1{,}000{,}000{,}000\ A_{45}$

The expected present value of the total death benefit payments to be made in the next five years is:

$$S = 1{,}000 \times 1{,}000{,}000 \times A^{1}_{45:\overline{5|}} = 1{,}000{,}000{,}000(A_{45} - v^5 \frac{l_{50}}{l_{45}} A_{50})$$

Therefore, the expected value of the fund at the end of 5 years is:

$(F_0 - S)(1+i)^5$ = 1,000,000,000 $\frac{l_{50}}{l_{45}} A_{50}$ = 1,000,000,000 $\times \frac{9,712.0728}{9,801.3123} \times 0.20508$ = 203,212,777

The development of the actual fund is as follows:

F_0 = 1,000,000,000 A_{45} = 1,000,000,000 × 0.15943 = 159,430,000

$F_1 = F_0$ × 1.065 = 169,792,950

$F_2 = F_1$ × 1.065 = 180,829,492

$F_3 = F_2$ × 1.065 – 1,000,000 = 191,583,409

$F_4 = F_3$ × 1.07 – 1,000,000 = 203,994,248

$F_5 = F_4$ × 1.07 = 218,273,845

Thus, the required difference between actual fund and expected fund is

218,273,845 – 203,212,777 = 15,061,608

[Total-10]

Solution 12 :

(i) As the compound bonuses vest before the payment of death benefit, the expected present value of the benefits is:

EPV benefit = 50,000 $\left[1.01923v\frac{d_{[40]}}{l_{[40]}} + 1.01923^2 v^2 \frac{d_{[40]+1}}{l_{[40]}} + \ldots\right]$

= 50,000 $\left[w\frac{d_{[40]}}{l_{[40]}} + w^2 \frac{d_{[40]+1}}{l_{[40]}} + \ldots\right]$

where w = 1.01923v = $\frac{1.01923}{1.06}$ = $\frac{1}{1.04}$ so that we can value the benefit as an endowment assurance using an interest rate of 4%.

i.e. EPV benefit = 50,000 $A^{4\%}_{[40]:\overline{20}|}$

Let P be the annual premium amount. The expected present value of the premiums is:

EPV premiums = P $\ddot{a}^{(12)}_{[40]:\overline{20}|}$

The expected present value of the expenses is:

EPV expenses = 0.5 P + 0.05 P $\ddot{a}_{[40]:\overline{20}|}$

Now, $A^{4\%}_{[40]:\overline{20}|}$ = $A_{[40]} - \frac{D_{60}}{D_{[40]}} A_{60} + \frac{D_{60}}{D_{[40]}}$

= $0.23041 - \frac{882.85}{2052.54}(0.4564-1)$ = 0.46423

$\ddot{a}_{[40]:\overline{20}|}$ = $\ddot{a}_{[40]} - v^{20}\frac{l_{60}}{l_{[40]}}\ddot{a}_{60} = 15.494 - \frac{1}{1.06^{20}} \times \frac{9,287.2164}{9,854.3036} \times 11.891$ = 11.9997

$\ddot{a}^{(12)}_{[40]:\overline{20}|}$ = $\ddot{a}_{[40]:\overline{20}|}$ - $\frac{11}{24}\left(1 - v^{20}\frac{l_{60}}{l_{[40]}}\right) = 11.9997 - \frac{11}{24}\left(1 - \frac{1}{1.06^{20}} \times \frac{9,287.2164}{9,854.3036}\right)$ = 11.6761

The equation of value is:

11.6761 P = 50,000 × 0.46423 + 0.5 P + 0.05 P × 11.9997

=> P = 2,194.73

Dividing this by 12, we find that the monthly premium is 182.89.

(ii) The expected present value of future benefits at the start of the eleventh policy year is:

EPV benefit = 50,000 $\left[1.01923^{11} v \frac{d_{50}}{l_{50}} + 1.01923^{12} v^2 \frac{d_{51}}{l_{50}} + \ldots\right]$

= 50,000 × 1.01923^{10} × $\left[w \frac{d_{50}}{l_{50}} + w^2 \frac{d_{51}}{l_{50}} + \ldots\right]$

where w = 1.01923v = $\frac{1.01923}{1.06}$ = $\frac{1}{1.04}$ so that we can value the benefit as an endowment assurance using an interest rate of 4% as before.

i.e. EPV benefit = 50,000 × 1.01923^{10} × $A^{4\%}_{50:\overline{10|}}$

The expected present value of future expenses at the start of the eleventh policy year is:

EPV expenses = 0.05 P $\ddot{a}_{50:\overline{10|}}$

The expected present value of future premiums at the start of the eleventh policy year is:

EPV premium = P $\ddot{a}^{(12)}_{50:\overline{10|}}$

Now, $A^{4\%}_{50:\overline{10|}} = A_{50} - \frac{D_{60}}{D_{50}} A_{60} + \frac{D_{60}}{D_{50}}$

$$= 0.32907 - \frac{882.85}{1,366.61}(0.4564 - 1) = 0.68024$$

$$\ddot{a}_{50:\overline{10|}} = \ddot{a}_{50} - v^{10}\frac{l_{60}}{l_{50}}\ddot{a}_{60} = 14.044 - \frac{1}{1.06^{10}} \times \frac{9,287.2164}{9,712.0728} \times 11.891 = 7.6946$$

$$\ddot{a}^{(12)}_{50:\overline{10|}} = \ddot{a}_{50:\overline{10|}} - \frac{11}{24}\left(1 - v^{10}\frac{l_{60}}{l_{50}}\right) = 7.6946 - \frac{11}{24}\left(1 - \frac{1}{1.06^{10}} \times \frac{9,287.2164}{9,712.0728}\right) = 7.4810$$

The gross premium prospective reserve at the start of the eleventh policy year is:

EPV future benefits + EPV future expenses – EPV future premiums

= 50,000 × 1.01923^{10} × 0.68024 + 0.05 × 2,194.73 × 7.6946 – 2,194.73 × 7.4810

= 25,574.12

[Total-12]

Solution 13 :

(i) Below table represents the growth of unit fund:

Year	Premium	Allocated Premium	Bid Offer spread	Policy Fee	Interest	Fund Before FMC	FMC	Fund at the end of the year
1	4500	4050	202.50	50	227.85	4,025.35	60.38	3,964.97
2	4500	4275	213.75	50	418.75	8,394.97	125.92	8,269.05
3	4500	4500	225.00	50	624.70	13,118.75	196.78	12,921.97

(ii) Below table represents the growth of non unit fund:

Surrender Penalty in Year 1 = 1200 × 10% × (1-0.001) = 119.880

Surrender Penalty in Year 2 = 750 × 5% × (1-0.002) = 37.425

Surrender Penalty in Year 13 = 0

Year	Allocation Charges	Bid Offer Spread	Policy Fee	Expenses and Commission	FMC	Interest	Death cover cost	Surrender Penalty	Net cashflows
1	450.00	202.50	50.00	1,175.00	60.38	(23.63)	0.991	119.880	(316.86)
2	225.00	213.75	50.00	140.00	125.92	17.44	4.135	37.425	525.40
3	-	225.00	50.00	140.00	196.78	6.75	9.691	-	328.84

(iii) Multiple decrement table is given by:

Age	qx	qs	aqx	aqs	apx	(t-1)apx
50	0.001000	0.10000	0.001000	0.099900	0.899100	1.000000
51	0.002000	0.05000	0.002000	0.049900	0.948100	0.899100
52	0.003000	-	0.003000	0.000000	0.997000	0.852437

Here, qx represents the mortality rate, qs represents the independent surrender rates

Year	Profit Vector	Probability in force	Profit Signature	EPV Profits	EPV Premiums
1	(316.86)	1.0000000	(316.86)	(293.39)	4,500.00
2	525.40	0.8991000	472.39	405.00	3,746.25
3	328.84	0.8524367	280.32	222.52	3,288.72

EPV Profits = -293.39 + 405.00 + 222.52 = 334.14

EPV of Premiums = 4500+3746.25+3288.72 = 11534.97

Profit Margin = 334.14/11534.97 = 2.897%

(iv) In the existing contract, occurrence of any surrender gives the company upfront profit in terms of surrender penalty. Also the surrender penalty in Year 1 as well as in Year 2 is greater than the expenses and commission payouts incurred by the company. Hence, surrender profit is positive.

With no surrender option, the company will defer the profit emergence. Also, the surrender profit will not occur. This will result into lower profit margin.

[Total-15]

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